

Volatility Magnification and Persistence in an Agent Based Financial Market

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Abstract

This paper presents a dynamic heterogeneous agent asset pricing model which is calibrated to aggregate financial time series. Its static and dynamic properties are compared with those of actual financial series. The setting, with many boundedly rational learning agents, is shown to replicate many features including magnifying volatility from an underlying stochastic dividend process. Most of the dynamics turn on how far back in time agents are willing to look to make optimal decisions in the present. Beyond this the paper explores the mechanism for increased volatility and finds that it may be connected with endogenous changes in agent heterogeneity, or market liquidity. Further empirical features are revealed which could be tested.

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1 Introduction

Modern financial markets still contain a large number of empirical features which remain a challenge for economic theories to explain.¹ Many of these results involve trying to line up relatively smooth macro economic time series with both the variability, and risk premia observed in financial markets.

Recent research has expanded beyond the general equilibrium, representative agent framework to explain many of these features.² The consideration of heterogeneous agents, some of whom may be less than completely rational is theoretically justified for several different reasons. First, the noise trader/limits to arbitrage literature emphasizes that risk averse rational traders may not be able to drive out irrational strategies.³ Second, it is important to remember that out of equilibrium the definition of rationality and optimality is less clear. Optimality is judged against the current population of strategies, not against fixed standard. This is related to the concept of coevolution in biology.⁴ Finally, from a robustness standpoint it is important to understand the underlying stability properties of rational expectations equilibria under learning.

This paper uses an agent based market which is closely related to LeBaron (forthcoming 2001*b*). Agent based models are controversial in terms of their theoretical usefulness and distinctions from previous heterogeneous agent models.⁵ Although there is no agreed on definition, agent based financial markets differ from other forms of heterogeneity in several ways. First, they are often populated with a larger number of traders and strategies. It is not uncommon to look at the interaction of thousands of different strategies in a market. Second, they are often more open ended in their approach to strategies and learning. Agents' strategies are less hard wired, and they are usually left to try to find their own best strategy using various learning mechanisms, many of which are drawn from artificial intelligence. Finally, and most importantly, heterogeneity in behavior is often endogenous in the system. Agents may or may not look similar depending on their recent learning dynamics. This is an important issue for financial markets since it allows for

¹See Campbell (2000*b*) for a recent summary.

²Several papers in this area are, Arthur, Holland, LeBaron, Palmer & Tayler (1997), Bak, Paczuski & Shubik (1997), Barberis, Shleifer & Vishny (1998), Beltratti & Margarita (1992), Brock & LeBaron (1996), Chiarella (1992), Cont & Bouchaud (2000), Daniel, Hirshleifer & Subrahmanyam (1998), Farmer & Joshi (forthcoming 2001), Frankel & Froot (1988), Gervais & Odean (1996), Hong & Stein (1999), Kirman & Teyssiere (1998), Levy, Levy & Solomon (1994), Rieck (1994), Youssefmir & Huberman (1997). Many have taken a very computational approach with large numbers of interacting and learning traders. A summary of some of the early work in this area is contained in LeBaron (2000).

³The evolutionary origins of this idea are in DeLong, Schleifer, Summers & Waldmann (1991). This runs counter to the old argument presented in Friedman (1953) that destabilizing strategies will be driven out of the market.

⁴The classic example of this in the social sciences is Axelrod (1984). An evolutionary computational experiment in social coevolution is Lindgren (1992).

⁵Even the name "agent based" is controversial. It is taken from computer science to refer to a system with a large number of autonomous, interacting agents. An argument can be made that this covers all economic modeling. The interested reader should see the web site of Leigh Tesfatsion (<http://www.econ.iastate.edu/tesfatsi/ace.htm>) for further examples and information.

endogenous liquidity, or the ability to find someone different to trade with. Agent based markets still share many features with other heterogeneous nonrational models. Agents are boundedly rational giving strategies which are more the result of an empirical search than heavy deductive reasoning. It is also possible that irrational strategies might be used at certain times, but they will still need to meet the evolutionary fitness test if they are to survive.

The form of irrationality considered in this market is important. Agents are assumed to be heterogeneous in the dimension of memory length, or the amount of past data used in decision making. Strategies are chosen based on how well they have performed over some past time period, where this is different for each agent. This can be interpreted that players forget what has happened in the distant past, but it also can play the role of representing a view that things have changed in the world, and old data are now irrelevant. This would correspond to recent discussions about the “new economy” in reference to asset pricing behavior. It also could correspond to a reasonable thing to do when time series contain actual changes in regime.⁶ Memory length also relates to a kind of learning about stationarity. While we explore many other types of learning in economics we rarely try to figure out how and when people are able to learn that the series they are observing are stationary.

The agent market is calibrated to U.S. aggregate financial series. The calibration remains slightly “loose” in that the model will not be a complete general equilibrium framework. Still, the calibration exercise is a useful one to assess the ability of this setup to fit various features. The model does well on many dimensions related to volatility, kurtosis, skewness, and predictability. However, it fails in a few interesting dimensions and these will be examined as well. Along with the calibration experiment, it is also shown that the model can converge to a well defined rational expectations equilibrium when agents are restricted to being very long memory.

At the core of this study is an attempt to understand what is causing persistence and amplification of volatility. This is done by analyzing many time series unavailable to researchers using actual market data. Although, the results are preliminary, they begin to shed some light on just how heterogeneity and learning can contribute to understanding empirical financial puzzles.

Section 2 gives a summary of the market structure. Section 3 presents the calibration results. Section 4 explores the volatility dynamics more fully, and section 5 concludes.

⁶See Pastor & Stambaugh (1998) for evidence on structural breaks, and Timmermann (1998) and Benink & Bossaerts (1996) for theoretical learning implications. Finally, Sargent (1999) shows the usefulness of a related type of learning using constant gain learning methods.

2 Market Structure

The market used here is based on LeBaron (forthcoming 2001*b*). More extensive calibration tests are presented in LeBaron (forthcoming 2001*a*).

2.1 Securities

The market is a partial equilibrium model with two securities, a risk free asset in infinite supply paying a constant interest rate, r_f , and a risky security paying investors a random dividend each period. It is available in a fixed supply of 1 share for the population, so if s_i is the share holdings of agent i the following constraint must be met in all periods,

$$1 = \sum_{i=1}^I s_i, \quad (1)$$

where I is the number of agents. The log dividend follows a random walk, with an annualized growth rate of 1.8 percent, and a annual standard deviation of 6 percent. This corresponds roughly to actual dividend properties from the U.S.⁷ The constant interest rate, r_f , is set to zero. This may seem extremely low since most studies set it to 1 percent. The purpose of setting the risk free rate to zero is to keep additional funds from coming into the system as bond holdings are increased. In the current market, new funds are only added through the dividend process. This turns out to be a useful benchmark, and is in the spirit of general equilibrium models.

Agents receive only two forms of income: dividends, and capital gains from purchases and sales of equity. These go to building wealth and current consumption. No other income streams are available.

2.2 Agents and rules

Agents are defined by intertemporal constant relative risk aversion preferences of logarithmic form,

$$u_{i,t} = E_t \sum_{s=0}^{\infty} \beta^s \log c_{i,t+s}, \quad (2)$$

subject to the intertemporal budget constraint,

$$w_{i,t} = p_t s_{i,t} + b_{i,t} + c_t = (p_t + d_t) s_{i,t-1} + (1 + r_f) b_{i,t-1}. \quad (3)$$

⁷See Campbell (forthcoming 2000*a*) for a summary of many of the features of aggregate financial series. Also, note that seasonalities in the aggregate dividend process are ignored.

$s_{i,t}$ and $b_{i,t}$ are the risky and risk free asset holdings respectively, and r_f is the risk free rate of return. r_t will represent the risky asset return at time t . These heavily restricted preferences are used for tractability. It is well known that for logarithmic utility the agent's optimal consumption choice can be separated from the portfolio composition and is a constant proportion of wealth,⁸

$$c_{i,t} = (1 - \beta)w_{i,t}. \quad (4)$$

To generate some additional agent diversity an idiosyncratic noise term is added to the consumption choice as in,

$$c_{i,t} = (1 + \gamma_i)(1 - \beta)w_{i,t}, \quad (5)$$

where γ_i is independent across agents and time, and is distributed $N(0, 0.16)$. Consumption is restricted to be less than wealth, and greater than zero. The noise term in consumption corresponds to individual liquidity demands for trading. Its main purpose is to maintain some small level of trading volume for cases in which the model is converging to a homogeneous equilibrium.

The time rate of discount, β , is set to $(0.95)^{1/12}$ which corresponds to 0.95 annual rate. This is a common time rate of discount used in macro models, and a later section will show that this provides a reasonable match for the equilibrium dividend yield in the model. It will be useful to denote the interest rate corresponding to the time rate of discount as r ,

$$r = \frac{1}{\beta} - 1. \quad (6)$$

A second property of logarithmic preferences is that the portfolio decision is myopic in that agents maximize the logarithm of next period's portfolio return. Agents will concentrate their learning efforts on this optimal portfolio decision. They are interested in finding a rule that will maximize the expected logarithm of the portfolio return from a dynamic strategy. The strategy recommends a fraction of savings to invest in the risky asset as a function of current information, z_t . The objective is to

$$\max_{\alpha_j} E_t \log[1 + \alpha_j r_{t+1} + (1 - \alpha_j)r_f], \quad (7)$$

⁸This result is well known in dynamic financial models. See Merton (1969) and Samuelson (1969) for early derivations. Altug & Labadie (1994) and Giovannini & Weil (1989) provide updated derivations in more general settings. Future versions of this model will generalize these preferences, but this brings in the added dimension of trying to determine optimal consumption given current information. Also related are the analytic policy rules for time varying returns in Barberis (2000) and Campbell & Viceira (1996). Finally, Lettau & Uhlig (1999) present some results showing some of the difficulties in building agents with the capabilities of learning dynamic consumption plans.

for the set of all available rules, α_j . In general it would be impossible for agents to run this optimization each period, since the above expectation depends on the state of all other agents in the market, along with the dividend state. The portfolio decision will therefore be replaced with a simple rule which will be continually tested against other candidate rules. This continual testing forms a key part of the learning going on the market.

The trading rules, α_j , should be thought of as being separate from the actual agents. The best analogy is to that of an investment advisor or mutual fund. A population of rules is maintained, and agents select from this set as they might chose an advisor. One difference here from the world of investment advisors is that the rule is a simple function, $\alpha(z_t; \omega_j)$, where z_t is time t information, and ω_j are parameters specific to rule j . The functional form used for each rule is a feedforward neural network with a single hidden unit with restricted inputs. It is given by

$$h_k = g_1(\omega_{1,k} z_{t,k} + \omega_{0,k}) \quad (8)$$

$$\alpha(z_t) = g_2(\omega_2 + \sum_{k=1}^m \omega_{3,k} h_k) \quad (9)$$

$$g_1(u) = \tanh(u) \quad (10)$$

$$g_2(u) = \frac{e^u}{1 + e^u}. \quad (11)$$

This framework restricts the portfolio to positive weights between zero and one, so there is no short selling or borrowing allowed. The model needs to set borrowing constraints to keep it off nonstationary bubble trajectories, and to avoid having to unwind debt positions when the agents go bankrupt. It is also important to note that this is a very restrictive neural network structure. Each hidden unit, h_k is connected to only one information variable. This differs from standard neural networks which let all inputs influence each hidden unit. This step was taken to enhance tractability of the learned rules, but it does limit the generality of how agents can combine input information. The weights are stored in a population table along with information on performance of this rule in the recent past. A simple real valued vector, ω_j , completely describes each dynamic trading strategy. All that is needed to be stored is a time series of the portfolio returns from each rule since the agent's objective only requires this as an input.

Agents chose the rule to use in the current period based on its performance in the past. They look over

their own past memory length, T_i , to evaluate the performance of the rule in the future using,

$$\max_j \hat{E}(r_p) = \frac{1}{T_i} \sum_{k=1}^{T_i} \log[1 + \alpha(z_{t-k}; \omega_j) r_{t-k+1} + (1 - \alpha(z_{t-k}; \omega_j)) r_f]. \quad (12)$$

The only feature driving heterogeneity across agents' decisions is their memory, T_i . This can lead to relatively similar decision rules, and very unstable markets. Further heterogeneity is added by making the rule decision have a random component. Agents compare their current rule to a candidate comparison rule drawn at random from the best half of active rules according to the agent's performance criterion. If this new rule beats the current one using the above return estimation, then it will replace the current one. If not, the agent continues to use the same rule. This might appear to give a very weak selection property for rules, but since agents get to have many chances to evaluate rules, they should slowly work their way to better and better strategies over time. Further heterogeneity is generated, by having only half the agents update their rules each period. This subset is chosen at random.

2.3 Information

Agents trading strategies are based on simple information structures which are input to the neural network, and used to generate the trading strategies, $\alpha(z_t; \omega_j)$. Obviously, the choice of the information set, z_t , is important. This set will be chosen to encompass reasonable predictors that are commonly used in real markets. The information set will include, returns, past returns, the price dividend ratio, and trend following technical trading indicators. In the current version, the only types of technical rules used are exponential moving averages. The moving average is formed as

$$m_{k,t} = \rho m_{k,t-1} + (1 - \rho) p_t. \quad (13)$$

Formally, the information set, z_t , will consist of 6 items.

1. $r_t = \log\left(\frac{p_t + d_t - p_{t-1}}{p_{t-1}}\right)$
2. r_{t-1}
3. r_{t-2}
4. $\log\left(\frac{r p_t}{d_t}\right)$
5. $\log\left(\frac{p_t}{m_{1,t}}\right)$

$$6. \log\left(\frac{p_t}{m_{2,t}}\right)$$

Several of the items are logged to make the relative units sensible. The dividend price ratio is normalized around a benchmark determined in the equilibrium presented in section 2.6. The two moving average indicators, $m_{1,t}$ and $m_{2,t}$, correspond to values of $\rho = 0.9$ and $\rho = 0.99$ respectively.

It is important to think about the timing of information as this will be important to the trading mechanisms covered in the next section. As trading begins at time t , all $t - 1$ and earlier information is known. Also, the dividend at time t has been revealed and paid. This means that α_j can be written as a function of p_t and information that is known at time t ,⁹

$$\alpha_j = \alpha_j(p_t; I_t). \quad (14)$$

All variables in I_t are known before trading begins in period t . p_t will then be determined endogenously to clear the market.

2.4 Trading

Trading is performed by finding the aggregate demand for shares, and setting it equal to the fixed aggregate supply of 1 share. Given the strategy space each agent's demand for shares, $s_{i,t}$, at time t can be written as,

$$s_{i,t}(p_t) = \frac{\alpha_i(p_t; I_t)\beta w_{i,t}}{p_t} \quad (15)$$

$$w_{i,t} = (p_t + d_t)s_{i,t-1} + (1 + r_f)b_{i,t-1}, \quad (16)$$

where $w_{i,t}$ is the total wealth of agent i , and $b_{i,t-1}$ are the bond holdings from the previous period. Summing these demands gives an aggregate demand function,

$$D(p_t) = \sum_{i=1}^I s_{i,t}(p_t). \quad (17)$$

Setting $D(p_t) = 1$ will find the equilibrium price, p_t . Unfortunately, there is no analytic way to do this given the complex nonlinear demand functions. This operation will be performed numerically. Also, it is not clear that the equilibrium price at time t is unique. Given the large number of nonlinear demand functions involved it probably is not. A nonlinear search procedure will start at p_{t-1} as its initial value, and stop at

⁹The impact of p_t on the current information vector, z_t , is taken into account as well.

the first price that sets excess demand to zero.¹⁰

It is important to remember the equilibrium is found by taking the current set of trading strategies as given. Once p_t is revealed then it is possible that agent i might want to change to a different trading rule. It is in this sense that the equilibrium is only temporary.¹¹

2.5 Evolution

Around this structure of rules, agents, and markets is an evolutionary dynamic that controls adaptation and learning in the entire system. Rules are evolved using a genetic algorithm.¹² This method tries to evolve the population using biologically inspired operators that take useful rules, and either modify them a little (mutation), or combine them with parts of other rules (crossover).

One of the crucial aspects of evolutionary learning is the fitness criterion which is used to select good parents from the current generation. It is not clear what makes a rule “fit” in a multiagent market. For example, it would be tempting to evolve the rules based on the historical performance on a fixed history of past data, but this would not capture the fact that agents are looking at different history lengths. To try to account for agent diversity a very weak selection criterion is used. A rule can be a parent for the next generation if at least one agent has used it over the last 10 periods. Rules that haven’t been used for 10 periods are marked for replacement. This is equivalent to eliminating all mutual fund managers with no customers.

Evolution proceeds as follows. First, the set of rules to be eliminated is identified. Then for each rule to be replaced the algorithm chooses between three methods with equal probability :

1. **Mutation:** Choose one rule from the parent set, and add a uniform random variable to one of the network weights, ω . The random increment is distributed uniform $[-0.25, 0.25]$.
2. **New weight:** Chose one rule from the parent set, chose one weight at random, and replace it with a new value chosen uniformly from $[-1, 1]$. This is the same distribution used at startup.
3. **Crossover:** Take two parents at random from the set of good rules. Take all weights from one parent, and replace one set of weights corresponding to one input with the weights from the other parent. This

¹⁰The search uses the matlab built in function `fzero`.

¹¹This is similar to the types of learning equilibrium surveyed in Grandmont (1998).

¹²The genetic algorithm, (Holland 1975), is a widely used technique in computational learning. Goldberg (1989) provides a good overview, and Mitchell (1996) gives a recent perspective. There are many evolutionary techniques, and this modified algorithm also contains inspiration from many of these others. Fogel (1995) provides a broad perspective to the complete set of methods.

amounts to replacing the two weights that affect the input directly (linear and bias), plus the weight on the corresponding hidden unit. Visually this is equivalent to chopping off a branch of the network for parent one, and replacing it with a branch from parent two.

A new rule is initialized by evaluating its performance over the past history of prices and information. Agents will use this performance history to decide on whether this rule should be used as they do with the others.

It would be difficult to argue that there is any particular magic to this procedure for evolving rules, and it goes without saying that these mechanisms are ad hoc. However, the objective is to produce new and interesting strategies that must then survive the competition with the other rules in terms of forecasting. Experiments have shown that the results are robust to different minor modifications of these mechanisms.

In contrast, agent evolution is not selective at all. A single agent is removed each period and replaced with one drawn from a common distribution for its memory length. The bond and equity holdings of the departing agent are distributed amongst the other agents, and the new agent is given an allocation of the median equity and bond holdings which is drawn from the others. In this way the agent changes are resource neutral in terms of its impact on the market. This process is designed to represent the random arrivals and departures of market participants. It also provides a weak dynamic moving the market in the direction of homogeneity.¹³

2.6 Equilibrium

It is useful in multi-agent financial simulations to have a benchmark with which the results can be compared. For multi-agent simulations the homogeneous agent world is often the appropriate benchmark. It turns out that in this model for the given calibrated parameters there is a homogeneous equilibrium in which agents all hold only the risky asset. Prices, dividends, and consumption all grow at the same expected growth rate which matches the rate for dividends. In this equilibrium stock returns should be unpredictable, and trading volume should be zero. In this sense it matches a classic efficient market situation where all information is contained in prices, and agents agree on functions mapping dividends into prices. Setting current consumption equal to dividends, and assuming all agents are the same gives the function mapping dividends to prices of

$$(1 - \beta)(p_t + d_t) = c_t = d_t \tag{18}$$

¹³Many experiments have been performed with agent evolution. For now this paper it was decided to stay with the simplest form. Removing agent changes completely has no basic impact on the results here. However, future versions will have to consider more detailed agent evolution to go with the learning and evolution in rule space.

$$p_t = \frac{\beta}{1-\beta} d_t = \frac{d_t}{r} \quad (19)$$

The second step of equilibrium determination is to show that for this pricing function it is optimal for agents to hold all savings in the equity security. This is done numerically using the dividend process, log preferences, and a range of portfolio choices.

The existence of such an equilibrium provides an important benchmark for the model. Given the complexity of agent based financial markets it is not simply enough to match empirical facts. The model should also be able to show that for some region of the parameters it can do something consistent with existing economic theory. The market can then be “taken out of the box” to perform more realistic studies of market dynamics.

2.7 Timing

The timing of the market is crucial since it is not an equilibrium model where everything happens simultaneously. A specific ordering must be prescribed to events. The following list shows how things proceed.

1. Dividends, d_t , are revealed and paid.
2. The new equilibrium price, p_t , is determined, and trades are made.
3. Rules are evolved.
4. Agents update their rule selection using the latest information.
5. Agents are evolved.

While this appears to be a sensible ordering, it is not clear if other sequences might give different results. The fact that there needs to be an ordering is limited by usual computing tools. The best situation would be for things to be happening asynchronously, and software tools are becoming available to tackle this problem. However, this would open some very difficult problems in terms of trading and price determination.

2.8 Initialization and parameters

While this market is intended to be relatively streamlined, it still involves a fair number of parameters which may not have as much economic content as one would like. The first of these are the initialization parameters which control the agent and rule structure at startup. Rules are started with parameters, ω , drawn from a

uniform $[-1, 1]$ distribution for each neural network weight. Agents begin with a memory, T_i , drawn from a specified distribution which will be set differently in various experiments. Bond holding levels are set to 0.1. The shares are divided equally among all agents. Finally, initial price and dividend series are generated using the stochastic dividend process, setting $p_t = d_t/r$.

There are several other parameters that will remain fixed in these runs, but which may be interesting to change in the future. The number of agents is set to 1000, the number of rules to 250, and the maximum history is set to 250. The period of inactivity after which a rule is deleted due to lack of use is fixed at 12 for all runs.

3 Results

The computer experiments presented here emphasize the key difference between two different cases. In the first, agents of many different memory lengths, T_i , are allowed to interact in the market. This is referred to as the *all memory* case. Initial agents are drawn from a uniform $[6, 250]$ distribution, and new entrants are drawn from the same distribution. This experiment is designed to explore the dynamics of the completely heterogeneous market setup. A comparison experiment, referred to as the *long memory* case, is provided by starting the market with a set of only longer memory agents drawn from a uniform $[200, 250]$ distribution.

3.1 Run Summaries

Figure 1 presents plots of prices and volume for the 10000 period run of the all memory case. Remember that periods are being calibrated to be one month in actual calendar time, so the 10000 period plot represents over 800 years of real price data. The figure shows a strong upward trend over the period which is due to the trending random walk of the underlying dividend process. It also displays a large amount of variability about this trend with some very dramatic dips, and sharp rises. The corresponding volume series shows a moderate amount of trading activity with turnover rates of nearly 5 percent per month. There also appears to be some clumping to volume activity along with a connection between volume and large price moves. These features will be demonstrated in future sections. The lower panel in figure 1 displays the wealth weighted average memory length over the run. The figure shows little deviation from the mean value of 127.5. If one expected a shift in wealth to longer memory agents, this would display an upward trend.

Figure 2 shows the same features for the long memory case. This displays a price series which appears to be following a random walk, and a volume series which is nearly zero. In the homogeneous equilibrium

the price series should be proportionate to the dividend series, and therefore follow a random walk as well. In the equilibrium all agents are in agreement on valuations, so trading volume should be zero except for a small amount driven by the idiosyncratic consumption demands. The simulation can occasionally generate some minor blips in trading as several agents may explore some out of equilibrium strategies, but they are quickly convinced to come back and join the rest of the crowd.

A dramatic comparison of the two cases is given by figure 3 which shows prices in the all memory case going through several large deviations from the trending random walk. Where as the long memory case shows little or no deviation from the simple trend. Figure 4 compares the continuously compounded (logged) returns including dividends in the two different cases. In the heterogeneous case there is clearly greater volatility, and several very large moves. The homogeneous case shows a much smaller amount of volatility, and very few large jumps in the return process. These same returns are plotted as histograms in figure 5. In both cases the histograms include a Gaussian distribution superimposed on the return distributions. In the top panel the all memory case displays strong deviations from Gaussianity which are typical of most financial series. The lower panel displays a distribution much closer to normality, and therefore different from actual return series.

3.2 Stock returns

Monthly continuously compounded excess returns with dividends are sampled from periods 5000 to 10000, and are given by

$$\log\left(\frac{p_t + d_t}{p_{t-1}} - r_f\right) \quad (20)$$

Sampling far out into the simulation run allows the system to move beyond the initial learning phase during which time some of the worst randomly initialized strategies are removed. Table 1 presents summary statistics for these returns in the two different cases along with comparison numbers for the S&P 500 index.¹⁴ The first four columns correspond to the series mean, standard deviation, skewness, and kurtosis. All three series show relatively similar mean monthly excess returns. The long memory case is actually the closest of the simulations to the actual data here. The most interesting value is the standard deviation. Here the all memory case shows a clear amplification of return volatility as compared to the long memory case. There is an increase by nearly a factor of five from this benchmark. The volatility of the all memory case is much closer to the actual S&P volatility, although it does give a value slightly higher than the actual returns

¹⁴The S&P numbers are sampled from 1926-1998 and are taken from the Ibbotson data set.

process. The column labeled kurtosis shows that the all memory simulation and the S&P series generate excess kurtosis indicating some deviations from normality. Returns in the long memory benchmark case are close to normal. If anything, the all memory series generates too large a kurtosis level. The columns labeled Q-ratio present quantile ratio values. These give another measure of the distribution shape. They are ratio in the left and right tails of the distribution of the 25th to the 5th, and the 75th to the 95th quantiles respectively. For a Gaussian distribution these would be 0.41. The table shows values close to Gaussian for the long memory case, but deviating in the tails for the all memory and actual data. The deviations appear slightly stronger in both tails for the all memory simulation than the actual data.

Table 1: *Excess Return Summary Statistics*

	Mean	Std	Skewness	Kurtosis	Q-ratio(L)	Q-ratio(R)
All Memory	0.64	0.111	-3.23	86	0.25	0.24
Long Memory	0.57	0.021	-0.02	3.00	0.41	0.43
S&P	0.58	0.056	-0.43	11.1	0.35	0.38

Mean and Std. are the monthly mean and standard deviation of the returns series inclusive of dividends. Skewness and kurtosis are also estimated at the monthly horizon. Values for the S&P are the total return less the 30 day T-bill rate monthly from Jan 1926 through June 1998. Q-ratio is the ratio of the 25th to 5th quantile in the left tail, and the 75th to 95th quantile in the right tail. These values should be 0.41 for a Gaussian.

Information on return dynamics is presented in figures 6 and 7. The first, figure 6 summarizes the autocorrelation features of the monthly return series for the two simulated markets along with the S&P. All three show little evidence for strong autocorrelation with only a few slightly large values of about 0.1 coming from the all memory case. Figure 7 turns to volatility by reporting the autocorrelations of the absolute value of returns. This picture clearly shows the all memory case following the actual market data in generating large positive volatility autocorrelations. The long memory case generates no persistence to volatility. This is consistent with a picture of what appear to be near independent returns for this important benchmark. The all memory case exhibits much larger volatility correlations at lower lags, and a much sharper drop off than for the actual returns series. This is consistent with the visual display in figure 4 which shows very sharp brief periods of intense volatility.

3.3 Trading Volume

This agent based stock market generates trading volume series along with price series. In a less than efficient market these are just as important as returns in characterizing what is happening.¹⁵

Table 2 summarizes informational heterogeneity and trading volume in the different simulation runs. The amount of heterogeneity is represented by the average trading volume from 5000 to 10000. Also, the variance across agents of their desired share holdings as a fraction of wealth presents another part of picture of agent disagreement. This is reported in the column labeled “holding variance”. “Rule Switches” shows the average number of agents (out of 1000) switching rules each period. For the first two cases the long memory case shows dramatically lower values which are usually less than 10 percent of their corresponding levels in the all memory case. Rule switches falls, but the reduction here is closer to about 1/4. The final column reports a more traditional measure of heterogeneity, the wealth Gini coefficient. The value for the all memory case is roughly 10 times that for the long memory case.

Table 2: *Volume and Heterogeneity Summaries*

	Trading Volume	Holding Variance	Rule Switches	Wealth Gini
All Memory	0.0035	0.0070	75.42	0.181
Long Memory	0.0004	0.0001	20.00	0.018

All values are time series averages over periods 5000 through 10000. Volume numbers are turnover ratios. Holding variance is the time series average of the cross sectional variance of agents' desired wealth fractions in equity. Rule switches are the average number of rule switches each period. Wealth Gini is the Gini coefficient on agent wealth.

Figure 8 displays the autocorrelation for volume in the all memory, long memory, and IBM stock respectively.¹⁶ This figure shows a remarkable similarity between the persistence in the all memory case and IBM. The long memory simulations show no persistence in trading volume.

Figure 9 shows the cross correlation between trading volume and return volatility measured as the absolute value of returns. This is well known to be positive for contemporaneous volume and volatility. The graph displays this property for the all memory, and IBM series. The long memory case displays no cross correlations. The all memory case generates a larger positive correlation than that for IBM volume data.

¹⁵See Karpov (1987) for an early survey, and Gallant, Rossi & Tauchen (1992) for a more recent display of price/volume facts.

¹⁶IBM volume is sampled from 1985 to 2000 at monthly frequency. Volume is not detrended. It seems more appropriate to compare trading volume with an individual stock, than the aggregate trading volume in the market.

3.4 Dividends and consumption

Figure 10 displays the dividend yield for both the all memory and long memory cases. In a stationary equilibrium this value should be constant, since there is no change in the fundamental riskiness of the equity asset. This is very nearly the case for the heterogeneous simulation result. It displays a value which is almost constant.¹⁷ The upper panel, corresponding to the all memory case, shows a much more realistic picture with a highly variable dividend price ratio.

As a standard infinite horizon investment consumption model this market generates a consumption stream as well as financial market prices. This adds another interesting dimension with which to test the results. Table 3 gives a summary of some of the results for aggregate consumption from the model. Given that all the series are nonstationary, results are given for annualized growth rates determined from the monthly consumption series aggregated to quarterly frequencies. The table shows general agreement in mean growth rates which is not surprising given the calibration is done with actual data. What is interesting is the amplification in volatility in the all memory case. In particular aggregate consumption volatility for the all memory case is over 30 percent, but for the actual macro series it is only about 3 percent.¹⁸ Since consumption is proportional to wealth in the simple log consumption case it is easy to see why the increased financial market volatility is transferred directly into consumption volatility.

This is a very important counterfactual for the agent based approach to fitting macroeconomic facts. Even though the market is a good mechanism for magnifying fundamental volatility into stock prices, it is important to remember that part of the puzzle of financial markets is also that this volatility does not appear in other macro series. In order to match this feature it will be important to think about other aspects of the consumption decision making process. Mechanisms such as habit persistence, or some kind of lagged wealth estimation may be necessary.

Table 4 examines whether the trading agents are behaving optimally. The two rows report various results averaged over all agents taken at $t = 10000$. The first uses the agent individual memory lengths to determine how much past data to use. The second takes the averages for all agents using 250 months of lagged time series. The first column simply reports the expected return of the agents' actual dynamic strategies, r_p , relative to a buy and hold position, r_m . The second column reports the fraction of these values which are

¹⁷Historically, the yield on stocks is close to 5 percent per year.

¹⁸It should be noted that the rate of 6 percent for the long memory case is not surprising either. This is the volatility of the dividend process which is higher than the volatility of aggregate U.S. consumption. If the dividends had been lined up with consumption as in Mehra & Prescott (1985) then this would line up with the actual consumption variability.

Table 3: *Consumption Growth Rates*

	Mean	Std	ACF(1)
All Memory	1.71	37.63	0.01
Long Memory	1.72	6.00	-0.03
U.S.	1.77	3.26	-0.12

Annual consumption growth rates and variability. For the U.S. this comes from annual data and is measured from 1891-1995, and the values come from Campbell (forthcoming 2000a). For the market simulations quarterly series are aggregated from the simulated monthly consumption series, and multiplied by 4, and $\sqrt{4}$, to get the annual mean and standard deviations respectively. The correlations are quarterly.

positive. The next two columns look at the simple first order condition,

$$E((r_{m,t+1} - r_{p,t+1})u'(c_{t+1})) = 0. \quad (21)$$

They report the fraction of agents for which this value is less than zero using individual, and aggregate consumption respectively. In all cases this table presents a similar picture. Using individual memory lengths, agents do not appear to be behaving in a very irrational fashion. They are generally choosing rules that beat the market (from their perspective), and do not systematically violate their first order conditions. On the other hand, if they use the entire 250 days of past information, then it is clear that they are not behaving optimally. The message here is that the short memory length is keeping these agents from changing to more desirable rules. However, they are still not being pushed out of the market.

Table 4: *Portfolio Optimality*

	$E(r_p - r_m)$	Fraction $(r_p - r_m) > 0$	FOC(Ind C) < 0	FOC(Agg C) < 0
Own Memory	0.04	0.67	0.28	0.29
Memory = 250	-0.09 %	0.36	0.03	0.03

r_p is the return of the dynamic agent strategy chosen by each agent. r_m is the market return, or a buy and hold strategy on equity. FOC measures $E((r_{m,t+1} - r_{p,t+1})u'(c_{t+1}))$. Ind C uses individual while Agg C uses aggregate consumption respectively.

The optimality of the different agent types is an important consideration in analyzing the feasibility and persistence of what might seem like irrational trader types. Their optimality relative to others is further analyzed in figures 11 and 12. The first compares estimated expected portfolio returns from period 9750 to 10000 for different agent types. These correspond to their own dynamic trading strategies over this period. The figure does not show the short memory types underperforming the long memory agents. On the contrary, the short memory types tend to be do quite well, or at least some subset is doing well. All that is clear

is that there is more variability for short memory agents which is sensible given greater variability in their objectives. The second figure, figure 12, looks at the mean log consumption over the same time period for the different agent types. It reveals a similar, though less dramatic, pattern in that there is little evidence for dominance by the long memory types.

All of these results on consumption and optimality are important. It would appear that outside of the homogeneous equilibrium, there is no indication that the longer memory types will dominate in terms of performance. It may be clear already that they are not taking over the market, but these results are important for other reasons. One could think of new entrants choosing memory lengths ex-ante to optimize their objective functions. The last two pictures suggest that it is clearly not obvious what choice they should make in this dimension. This strengthens the case for the continuing entry of short memory agents, since their irrationality is far from obvious.

4 Large Price Movements

The price and returns series shown display a market which can occasionally go through dramatic periods of volatility. The relatively stable fundamental series is somehow magnified into a much more variable price series through the dynamics of trading in the market. The results so far suggest that much of the cause for this variability in this market comes from short memory traders. This section analyzes the behavior of prices and traders around large moves in greater detail.

The following pictures examine the movements of various series around large price rises and price falls. Large moves are defined as being in the 0.005 tail of the distribution at either the high or low end. Other series are averaged in periods around the large price events as they would be in normal event studies.¹⁹ In each case the y-axis values represent the ratio of the mean values around the price change divided by their unconditional mean across the sample. 95% confidence bands are also presented.²⁰

Figure 13 shows the movements of volatility and trading volume around large price moves for the all memory run. Volatility is again measured as the absolute return. The pictures clearly show large increases in volatility and trading volume around price decreases and increases. Trading volume looks slightly different on a price fall where it exhibits a slight reduction just before the price drop. While most of these figures will cover the all memory case, figure 14 presents a comparison with the benchmark long memory case. It

¹⁹There can be overlap between large price events. This is left in the samples.

²⁰Given the sample overlap these should be viewed with some caution. They are presented to give the reader some idea of significance.

clearly shows no interesting dynamics. There is no evidence for volatility or volume persistence, and there is complete symmetry from the price increase and decrease cases. This is as should be expected if the return series are independent over time for this case.

Figure 15 looks more directly at agent heterogeneity. It first examines the fraction of rules that the agents are currently using. This is referred to as “rule dispersion”. Since the number of rules in use can vary over time this is a good proxy for heterogeneity and market liquidity. It appears to be generally large around a crash, but there is a brief drop just before a crash itself. This drop is repeated in the second measure, “holdings variance”, which reports the variance over the agent wealth fractions in equity. This second difference of opinion measure reports a similar drop just before a price drop. Equivalent features on price increases appear to not be present.

The top part of figure 16 reports a similar feature in the dynamics of rule switching. It is intuitively clear that rule switching increases around large price moves. Agents chose these periods to change to another better performing rule since the price variability increases the chances that other rules may beat currently used rules either by chance or by skill. The bottom part of figure 16 looks at one part of the conjecture that wealth under the control of shorter memory agents may cause the large moves. It looks at inequality in the wealth levels through the Gini coefficient. Large price moves cause inequality to increase, but there is no evidence for extra inequality before a crash.

Figure 17 shows the dividend growth rate around extreme price changes. This figure is striking in its lack of any pattern. There appears to be no discernable connection from dividend movements into the eventual large price changes. The bottom panels continue by analyzing the changes in the d/p ratio around large price change. In the case of a large price fall the d/p ratio rises as it should, but in the case of a rise the change is less dramatic and sudden. The d/p ratio itself can be used as a possible indicator that a crash is eminent, and it is used in figure 18. This picture looks at moves around extremes both low and high in the d/p ratio for dispersion and rule variances. In both cases the sharp drop off in dispersion near crashes is very clear, although the drop off in holdings variance may not exactly precede a sharp price drop. Figure 19 repeats this experiment for volatility and trading volume. Interestingly, it also shows reductions in volume and volatility just before a crash measured as lows in d/p ratios. The results are very different for high d/p ratios.

The final plot, figure 17, shows the dynamics of two other variables. The first, labeled “memory”, represents the average memory length weighted by wealth. This shows a very weak indication that wealth is more concentrated at short memories before a crash, but it is a very disperse index with a very weak pattern.

The final figure reports agent bonds holdings. It shows a dramatic drop off just before a crash. This pattern may indicate a final rush to get fully invested during a price run up, or it may indicate the possibility of a crash increases as bond holdings are reduced. The latter effect could be caused by the fact that agents are under more pressure to sell their equity portfolios when they have no bond holdings from which to finance consumption shocks.

5 Conclusions

The results in this paper show that an agent based model is capable of quantitatively replicating many features of actual financial markets. Comparisons show favorable results for returns and volatility and their persistence. The data also replicated the well known feature of excess kurtosis, or too many large moves, in the returns series. It also was able to generate pictures of volume/volatility cross correlation. Given the market is forced to rely on a dividend process fit to the U.S. aggregate, and to keep within the bounds of well defined, restrictive, intertemporal preferences these successes are quite remarkable. Finally, the same model populated with only long memory types converges to a well defined equilibrium and behaves in a way which is consistent with an efficient market.

The market also demonstrates some interesting and important deviations as well. The most important being the extreme volatility in the consumption series induced by the fact that consumption wealth ratios are constant for log preferences. This important issue from macro finance is clearly not solved here. Further extensions will be necessary to try to replicate this fact. Possible changes might be to add nondiversifiable income streams, or habit persistent preferences. Probably most consistent with this framework might be to set up consumption rules that based on weighted averages over lagged wealth levels rather than current ones. This would give a lagged response from asset price movements to the consumption process.

It was also demonstrated that agent behavior when measured according to their own memory lengths did not show strong one sided biases from rationality. A strong majority of agents favored their dynamic portfolio holdings to buy and hold strategies when sample at a single point in time. Furthermore, observed consumption and return series do not appear to favor long memory agents. There is some weak evidence for greater cross sectional variability for short memory agents, but there is no strong evidence that the long memory types are doing better. This is important given that one might consider memory length as a choice parameter that people can pick when they enter the market and chose strategies.

The large price moves are explored in some detail to understand what drives the magnification and

persistence of volatility. The figures show some common features to many of the observed measures, including strong differences between price increases and decreases on many of the series. There is also an indication that market heterogeneity falls just before a large fall in prices. This could be an example of market liquidity drying up as agents all shift to common successful strategies. This drop shows up in both trading volume, and in other internal measures of heterogeneity. It remains an interesting question if this feature appears in real markets.²¹ Unfortunately, direct evidence for the impact of shorter memory agents on large moves is not strong.

This market should be viewed as part of an ongoing process of using large scale heterogeneous agent learning models to explore features from finance and macroeconomics. The goals are to emphasize the importance of heterogeneity in modeling financial markets, and to explore how differences of opinion and strategies affect the learning process itself. While these models themselves should play a useful role, they may also be used as exploratory tools for more traditional theoretical approaches to heterogeneous agent setups. In all cases coevolution and endogenous heterogeneity will play crucial roles toward understanding economic behavior.

²¹Chen, Hong & Stein (2000) verify certain features from a heterogeneous agent market in actual volume and returns data. Direct tests such as these will be necessary in validating these models.

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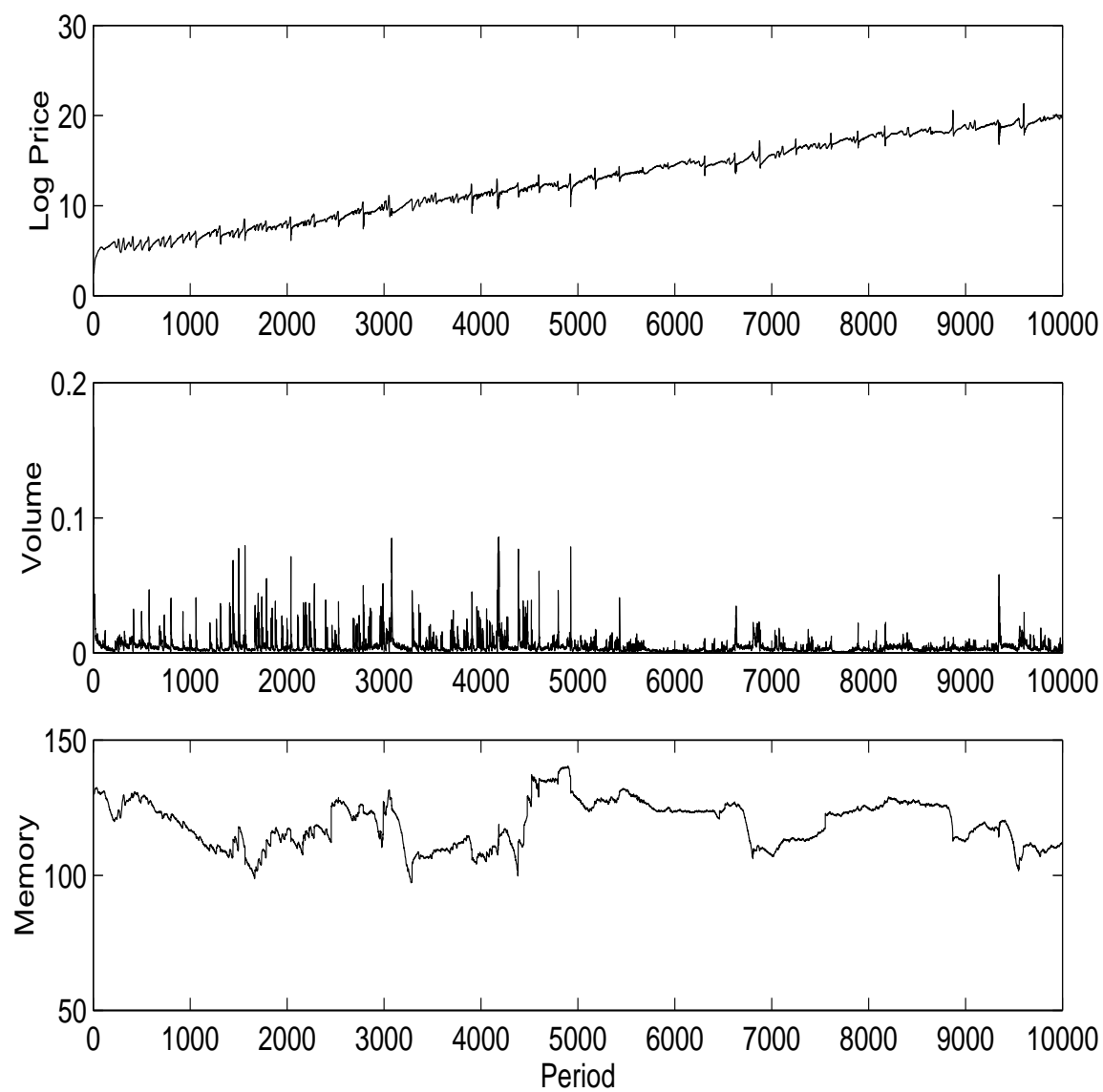


Figure 1: **Price//Volume/Memory: All Memory**

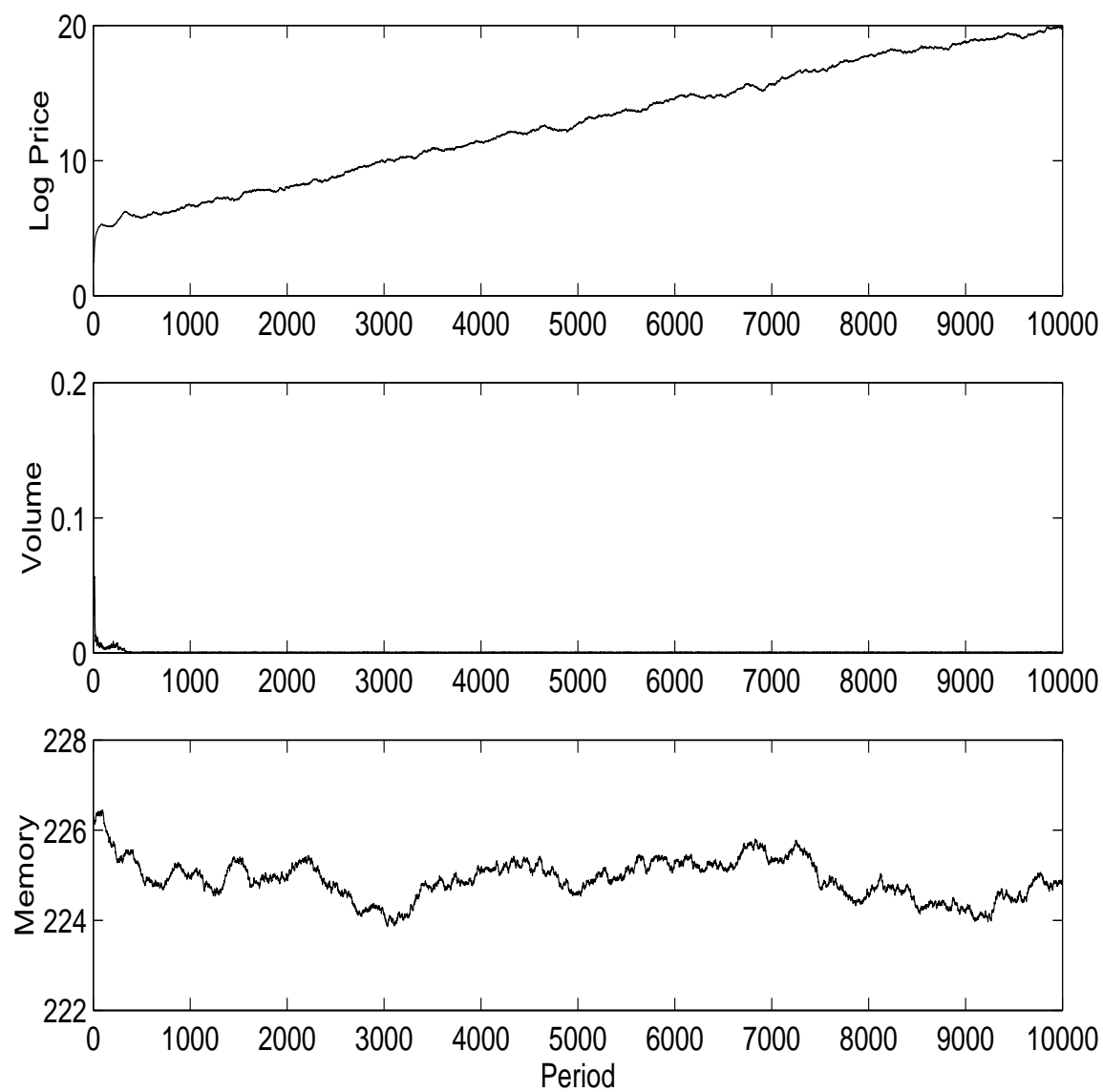


Figure 2: **Price/Volume/Memory: Long Memory**

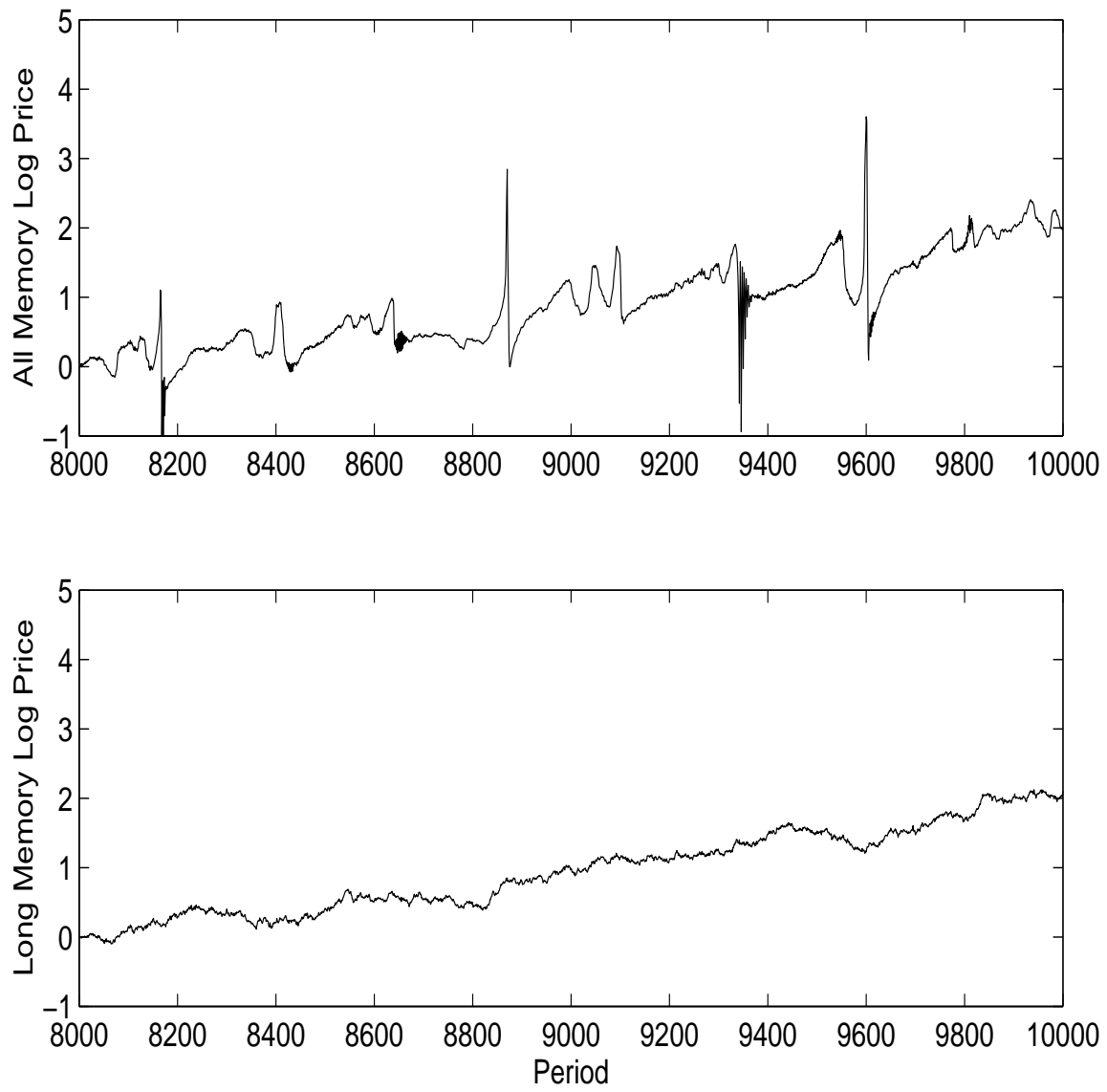


Figure 3: **Price Series Comparison: All/long Memory**

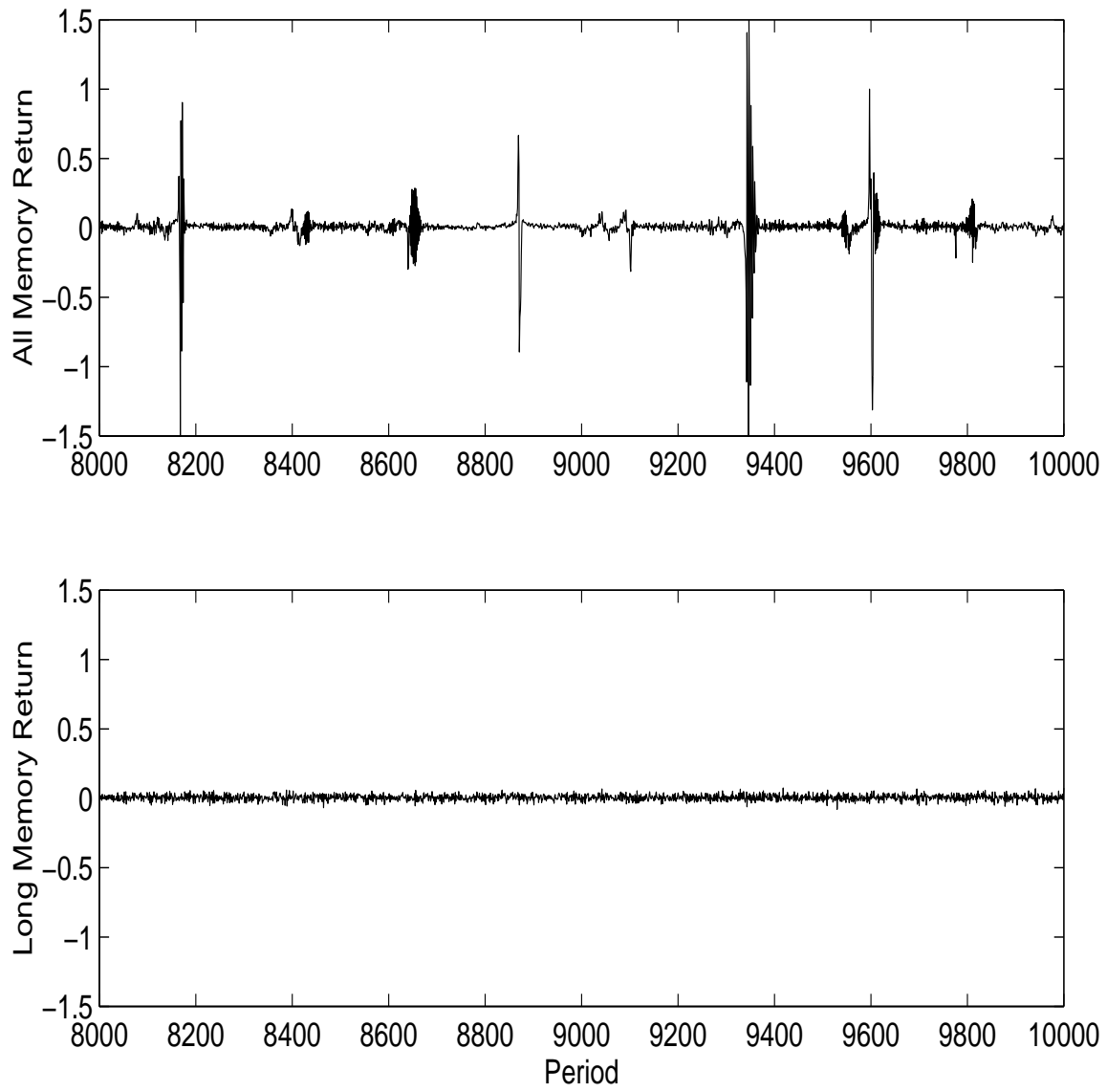


Figure 4: **Return Series Comparison: All/long Meomry**

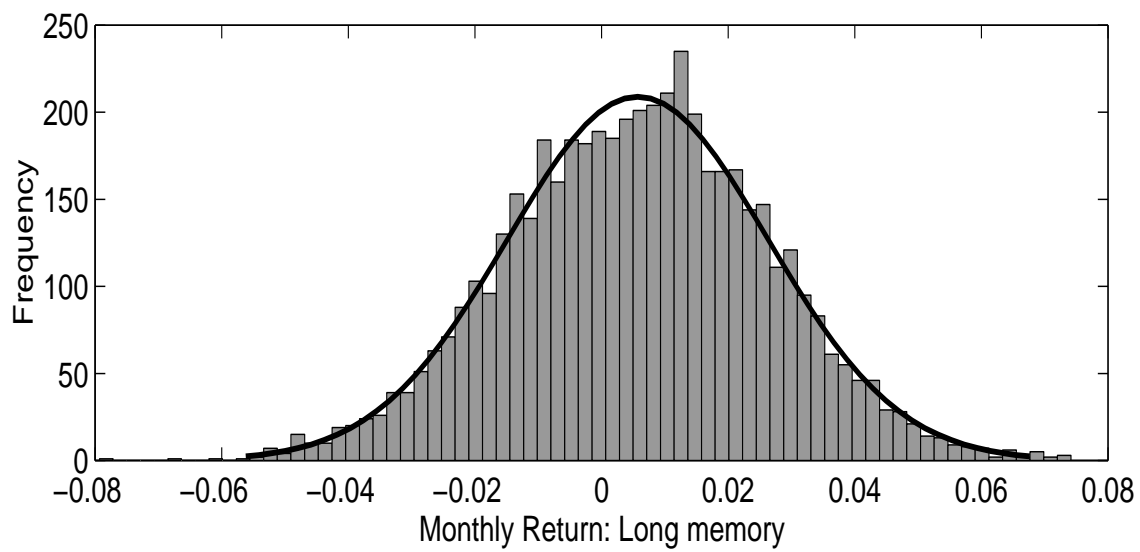
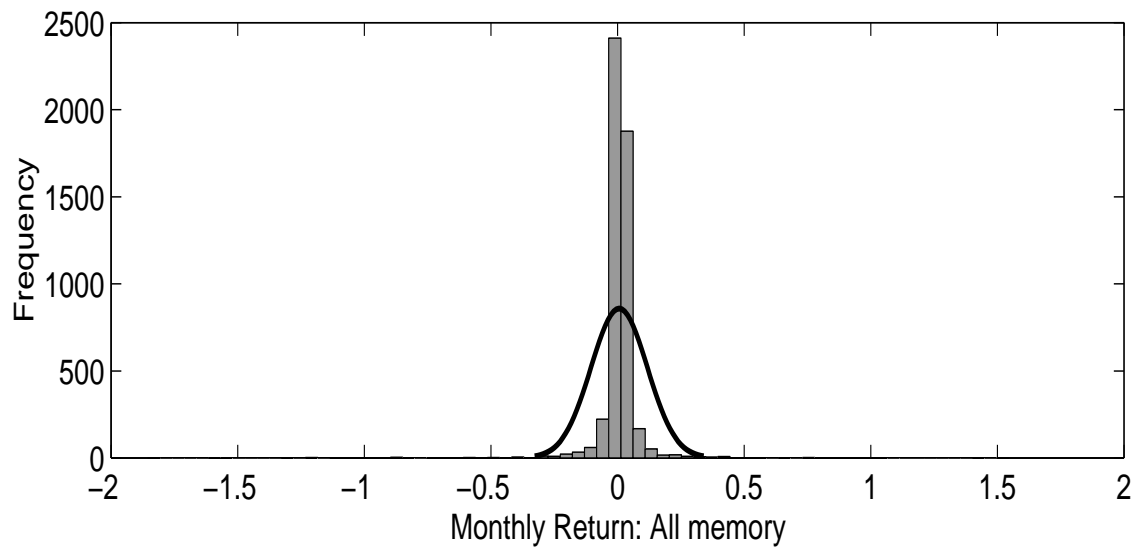


Figure 5: **Return Distribution Comparison**

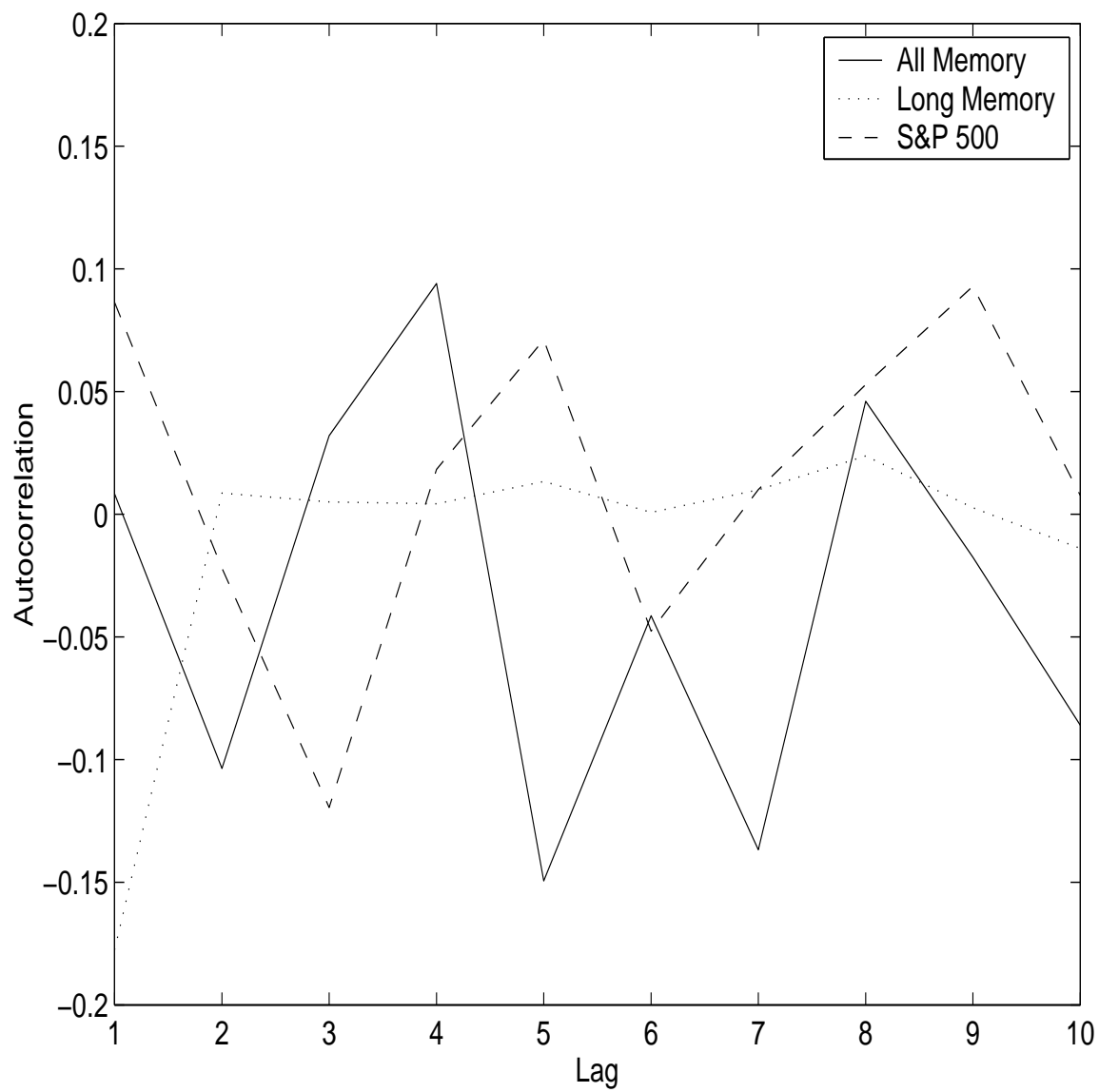


Figure 6: **Return Autocorrelations**

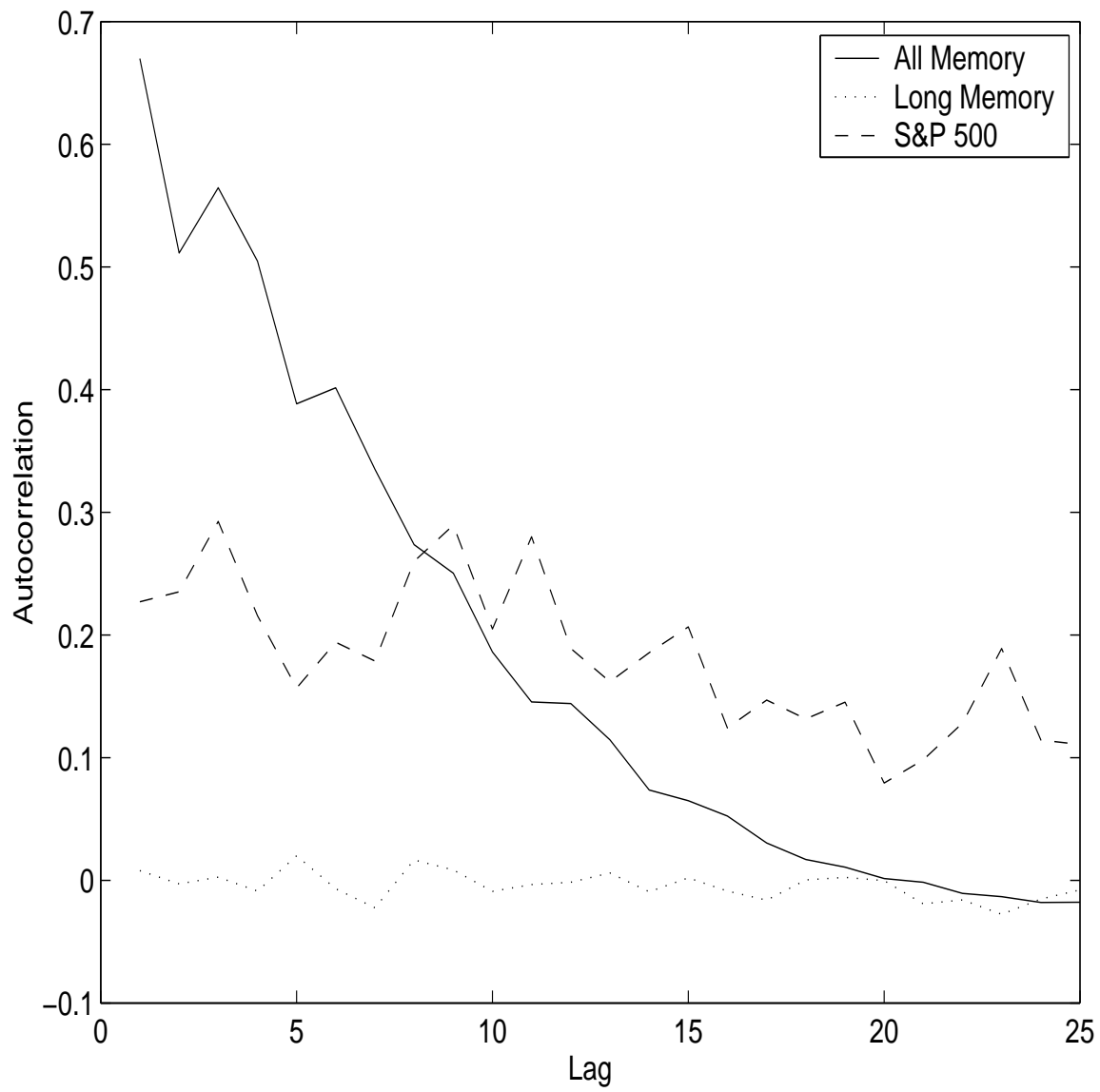


Figure 7: **Absolute Return Autocorrelations**

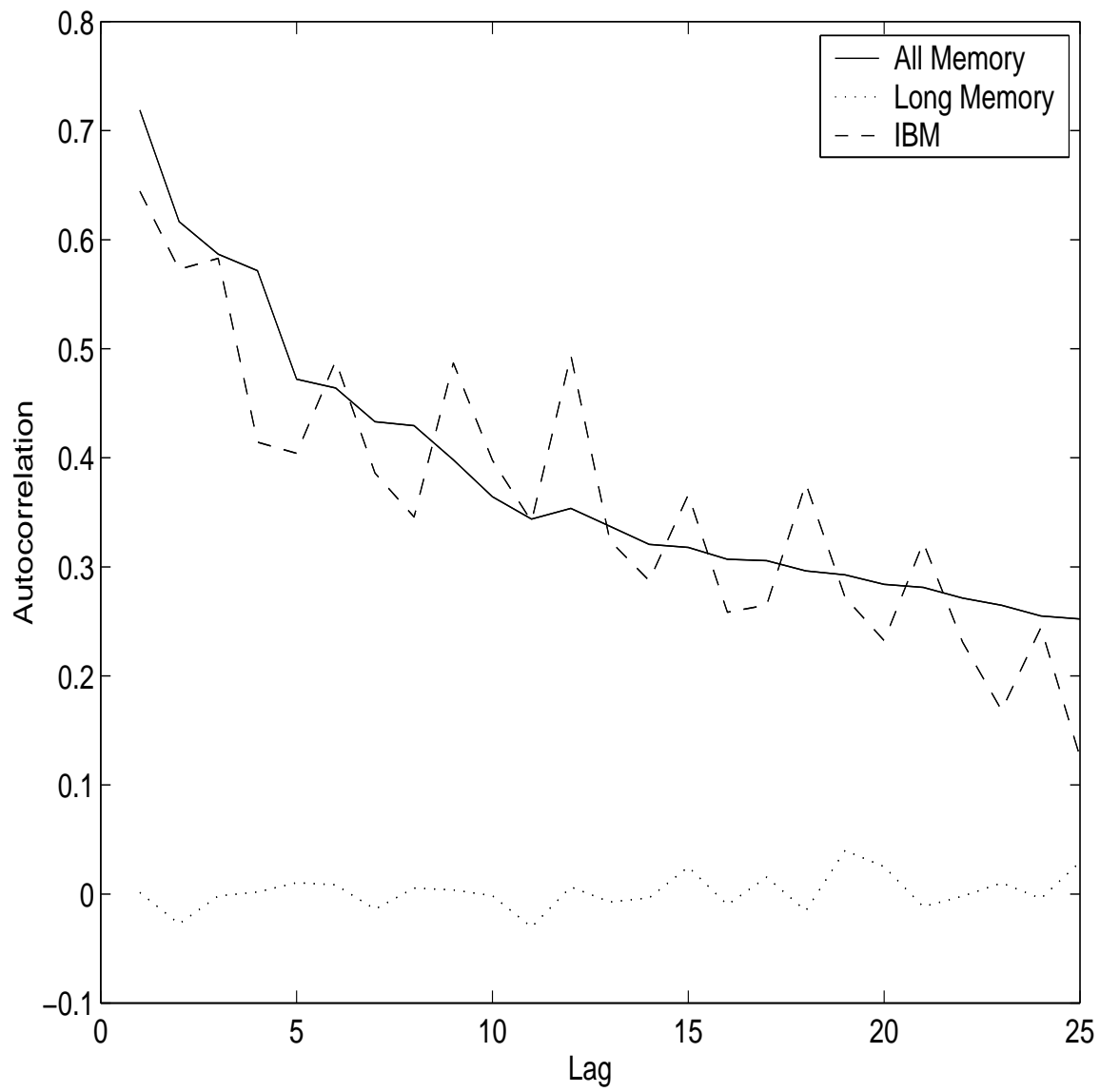


Figure 8: **Trading Volume Autocorrelations**

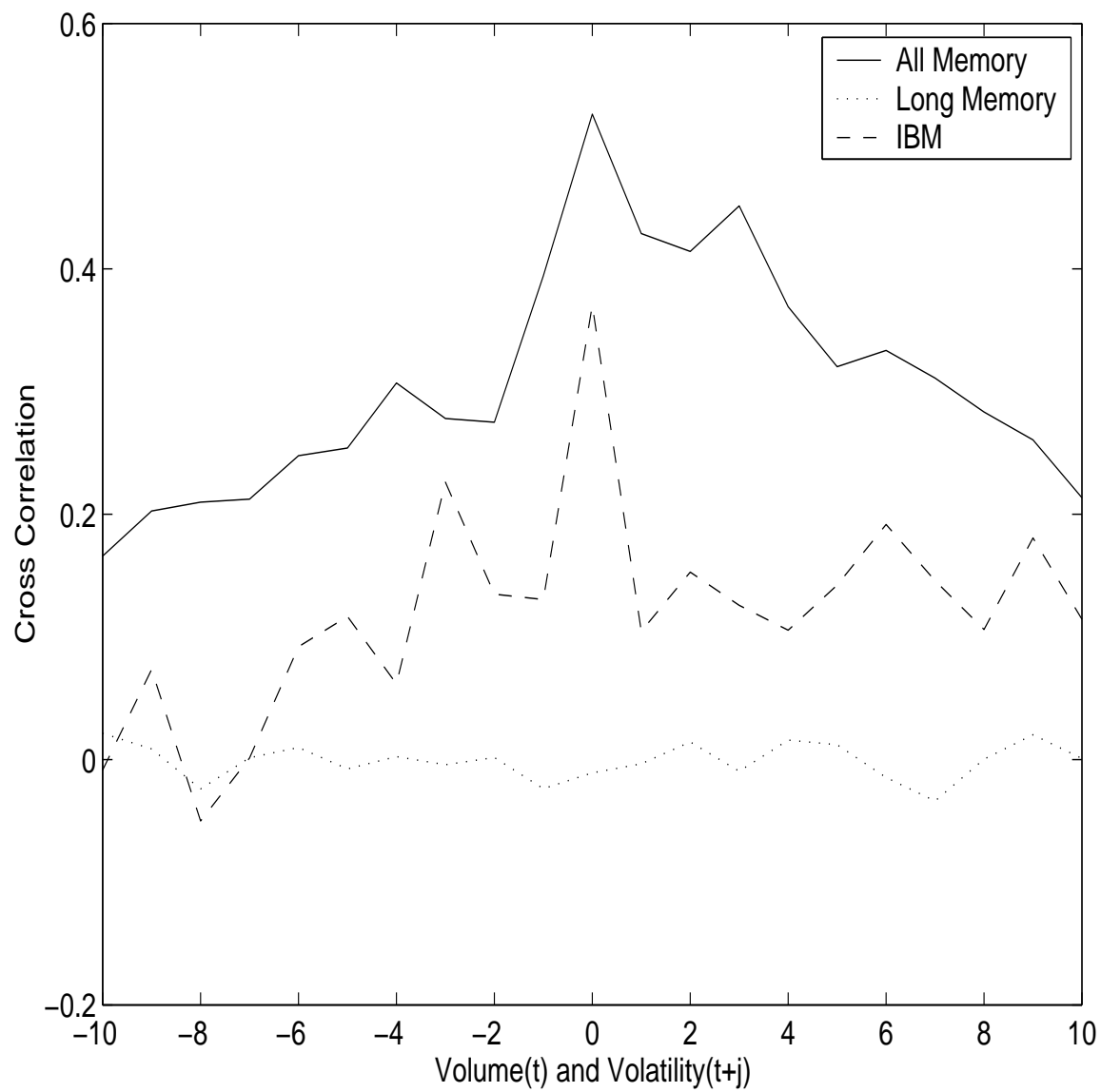


Figure 9: **Volume Volatility Crosscorrelations**

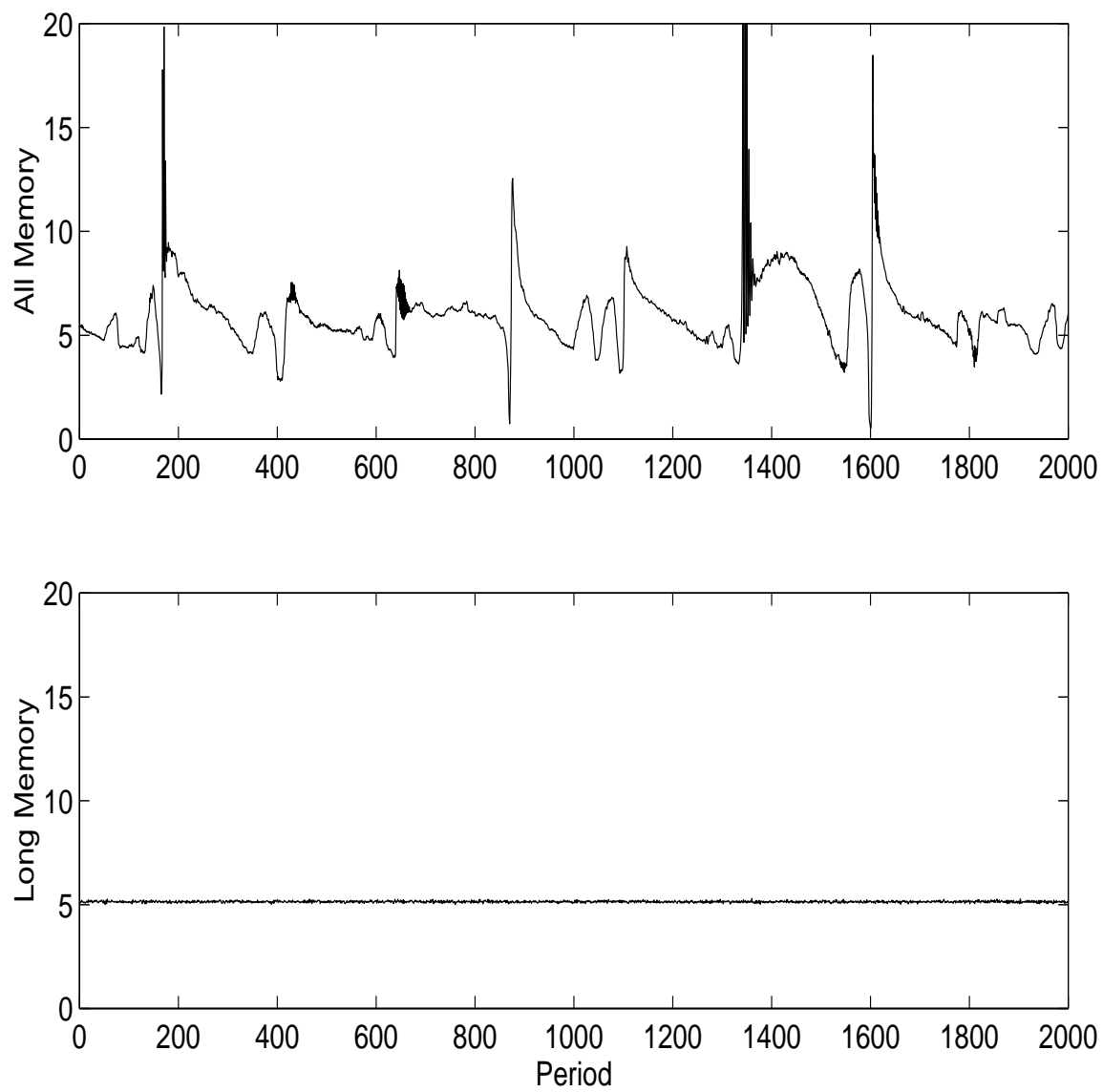


Figure 10: **Percentage Dividend Yield (d/p) (Annualized)**

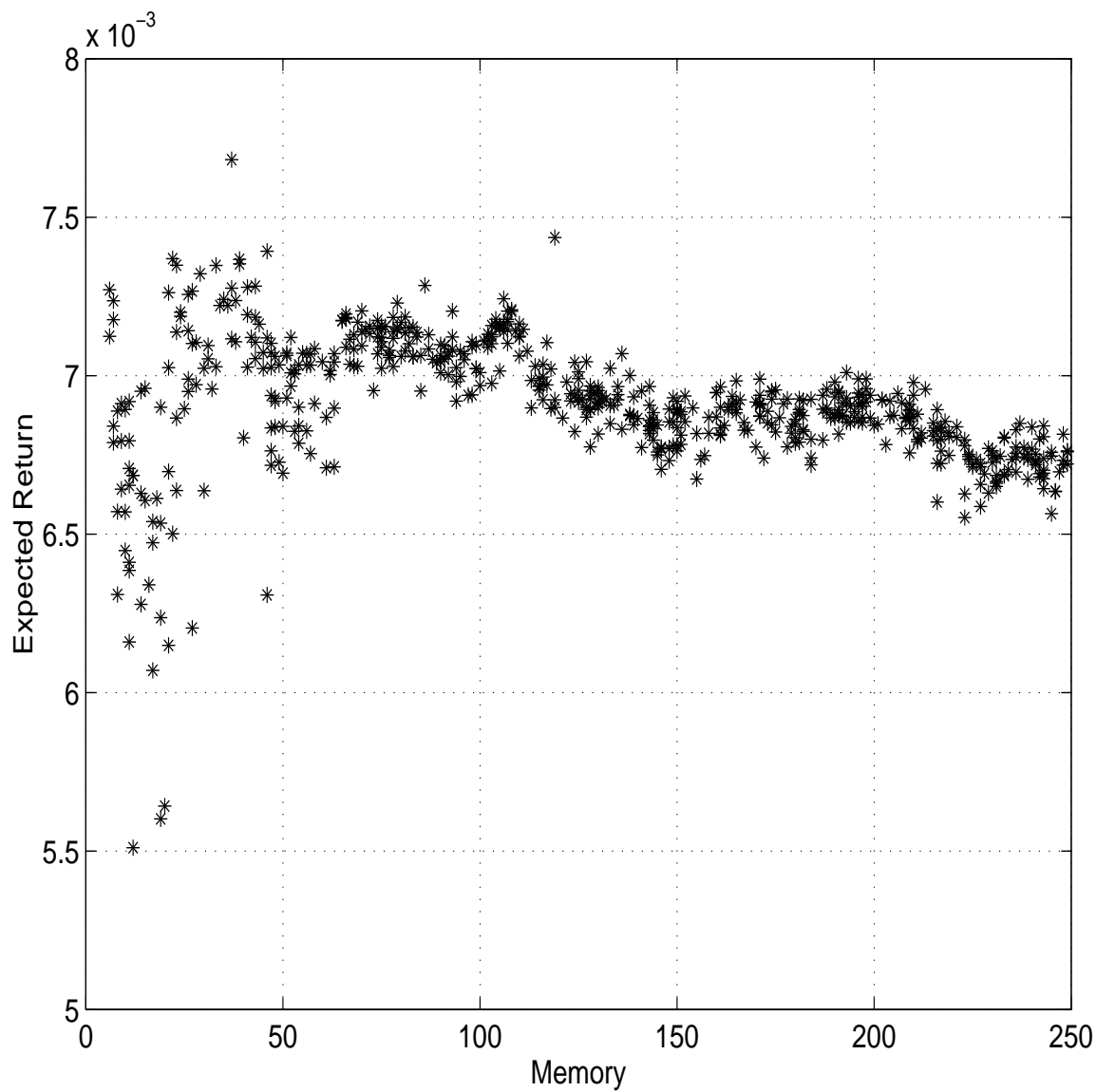


Figure 11: **Memory Versus Expected Return**

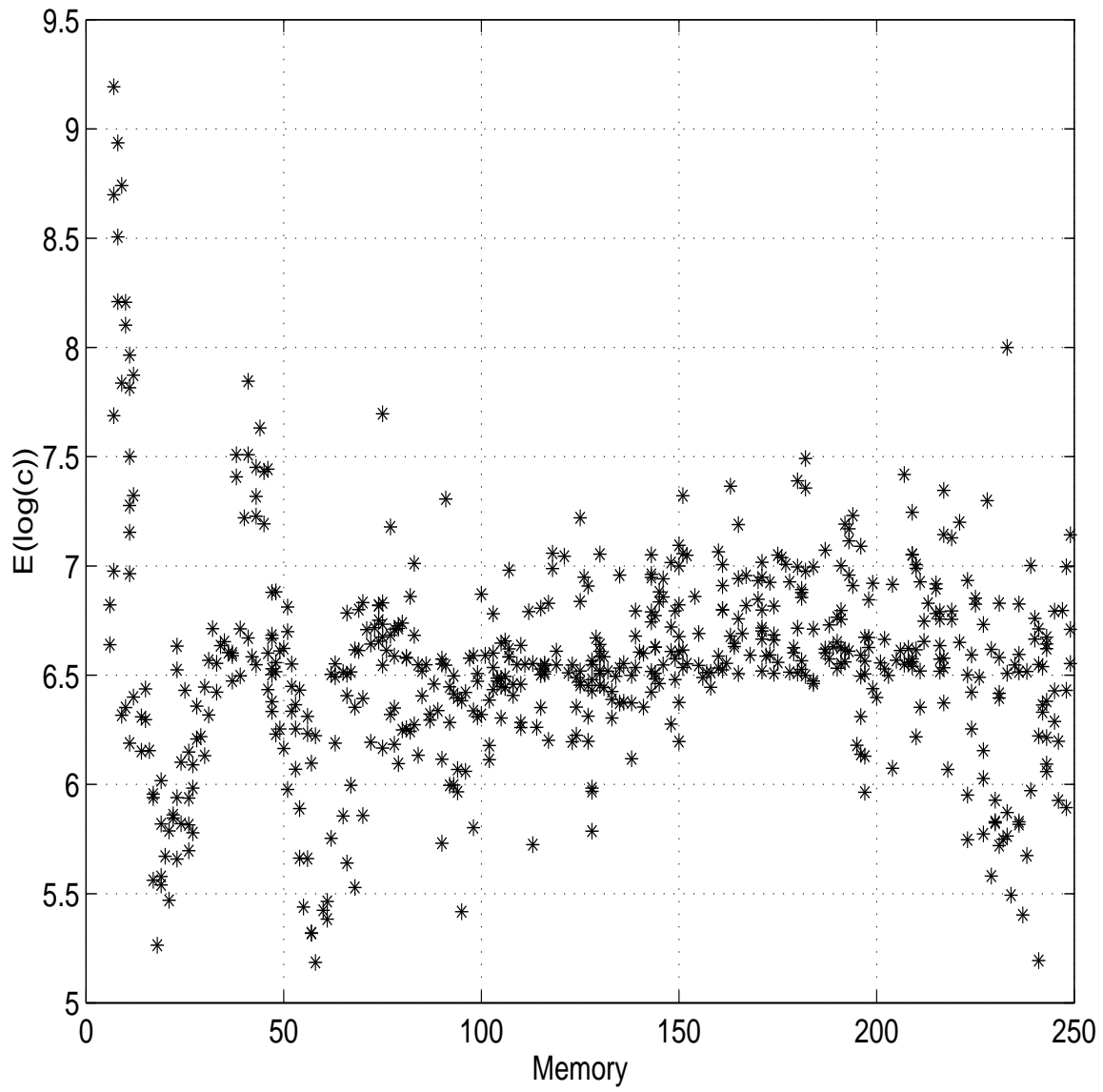


Figure 12: **Memory Versus $\log(c)$**

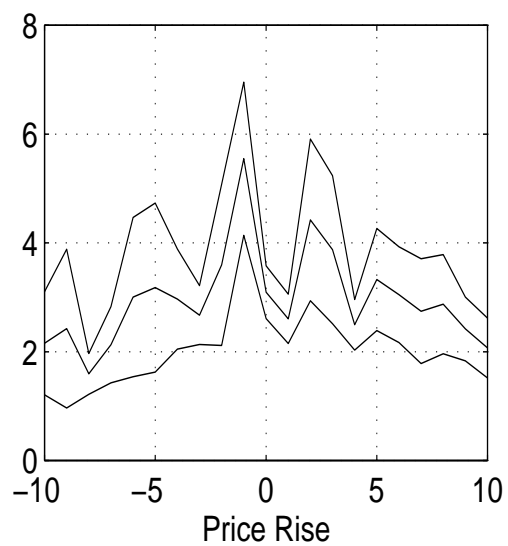
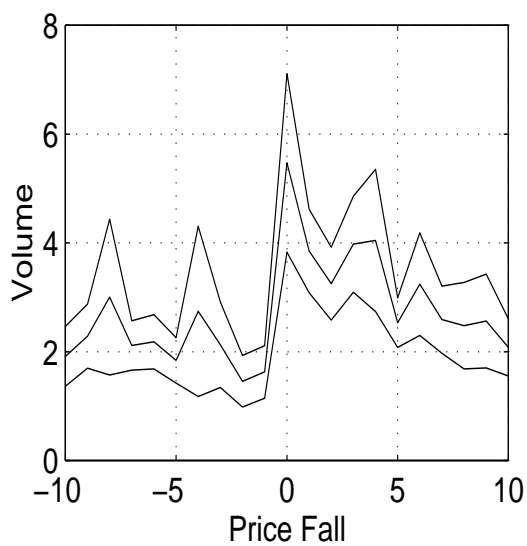
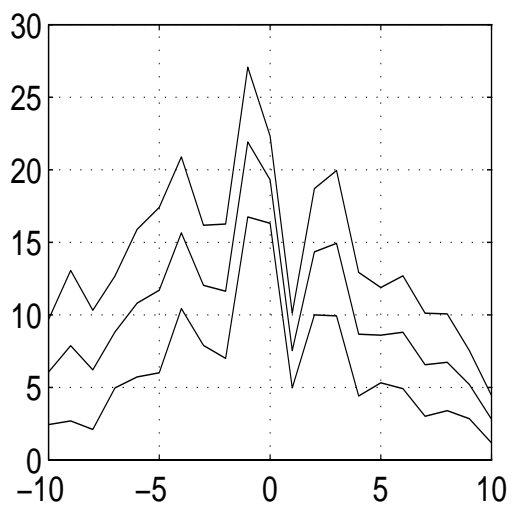
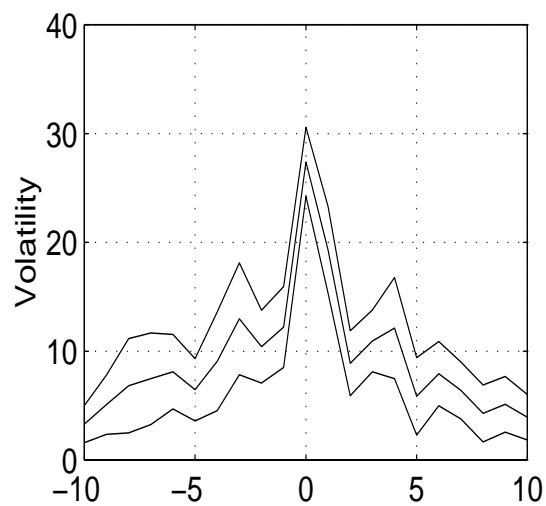


Figure 13: **Price Jumps: Volatility and Volume**

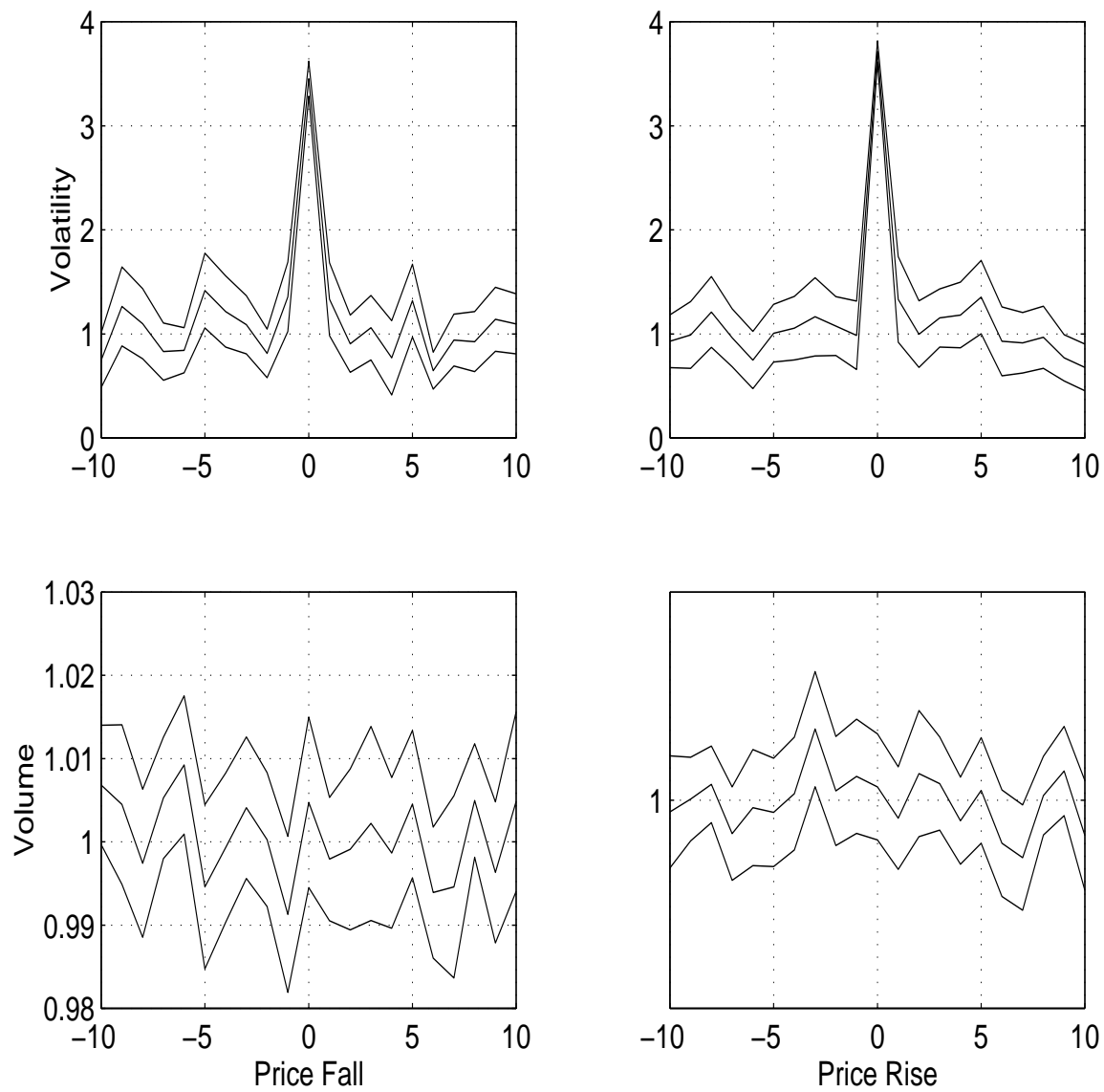


Figure 14: **Price Jumps: Volatility and Volume (Long Memory)**

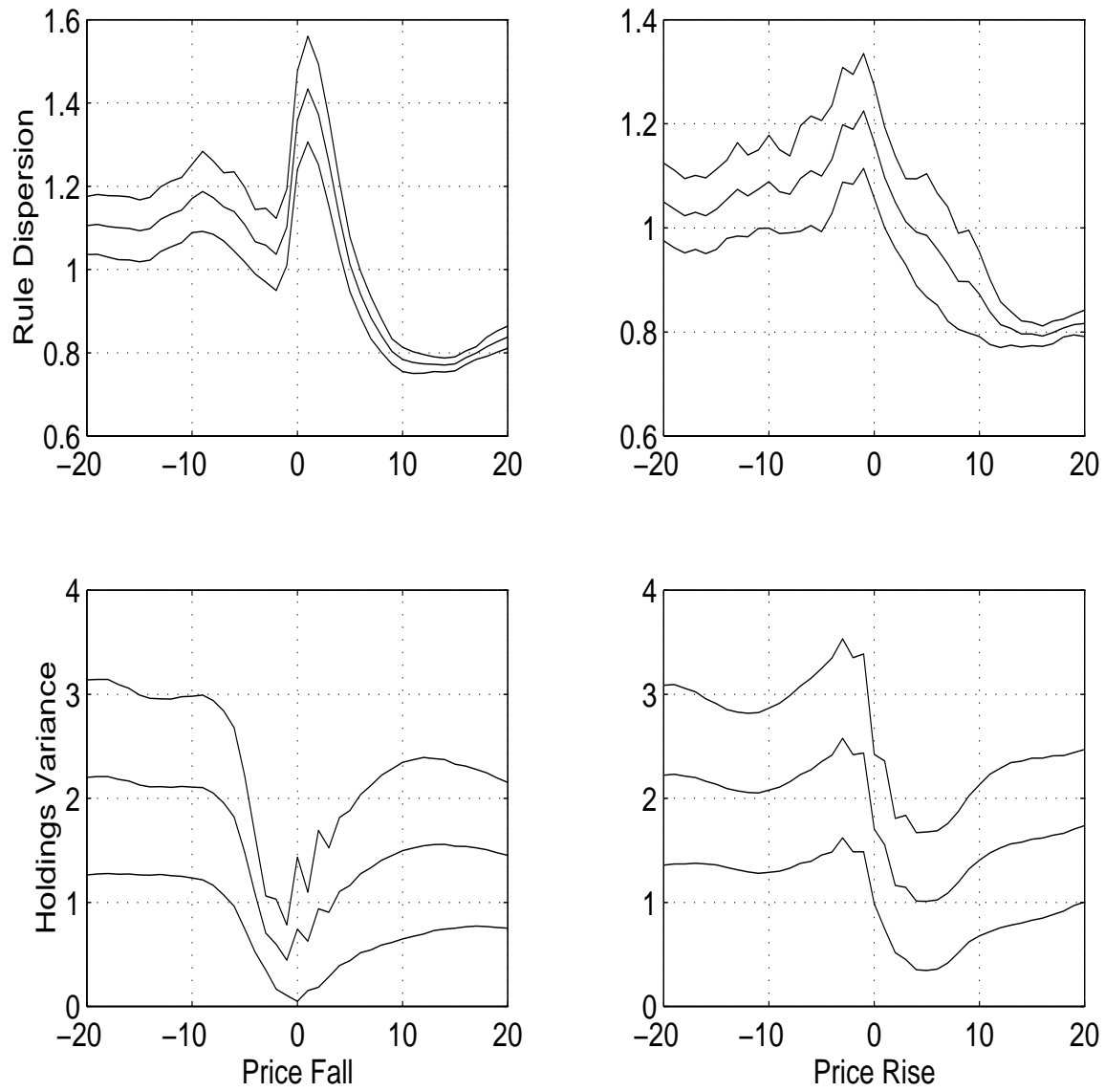


Figure 15: **Price Jumps: Dispersion and Holdings Variance**

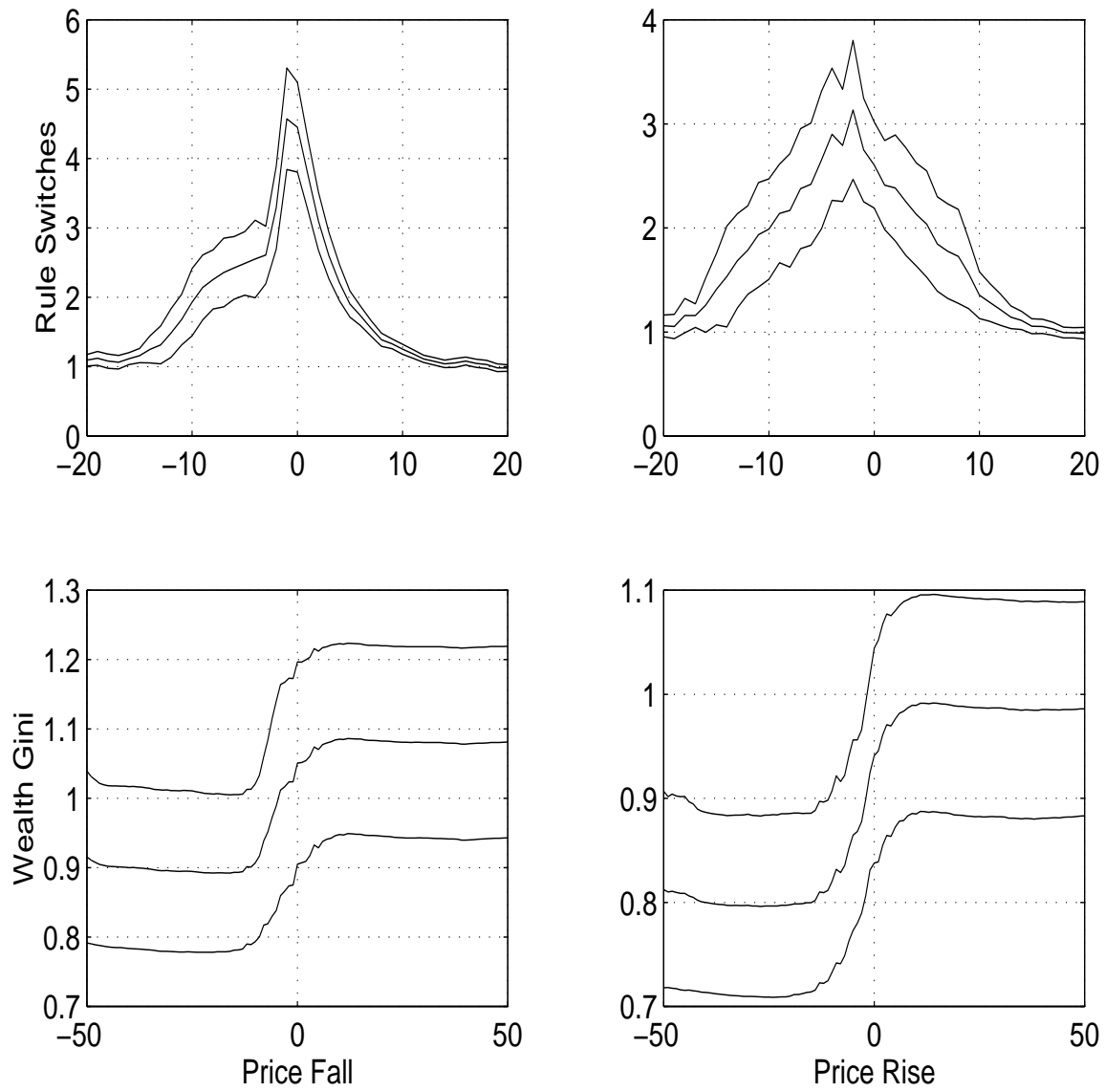


Figure 16: **Price Jumps: Rule Switches and Wealth Gini**

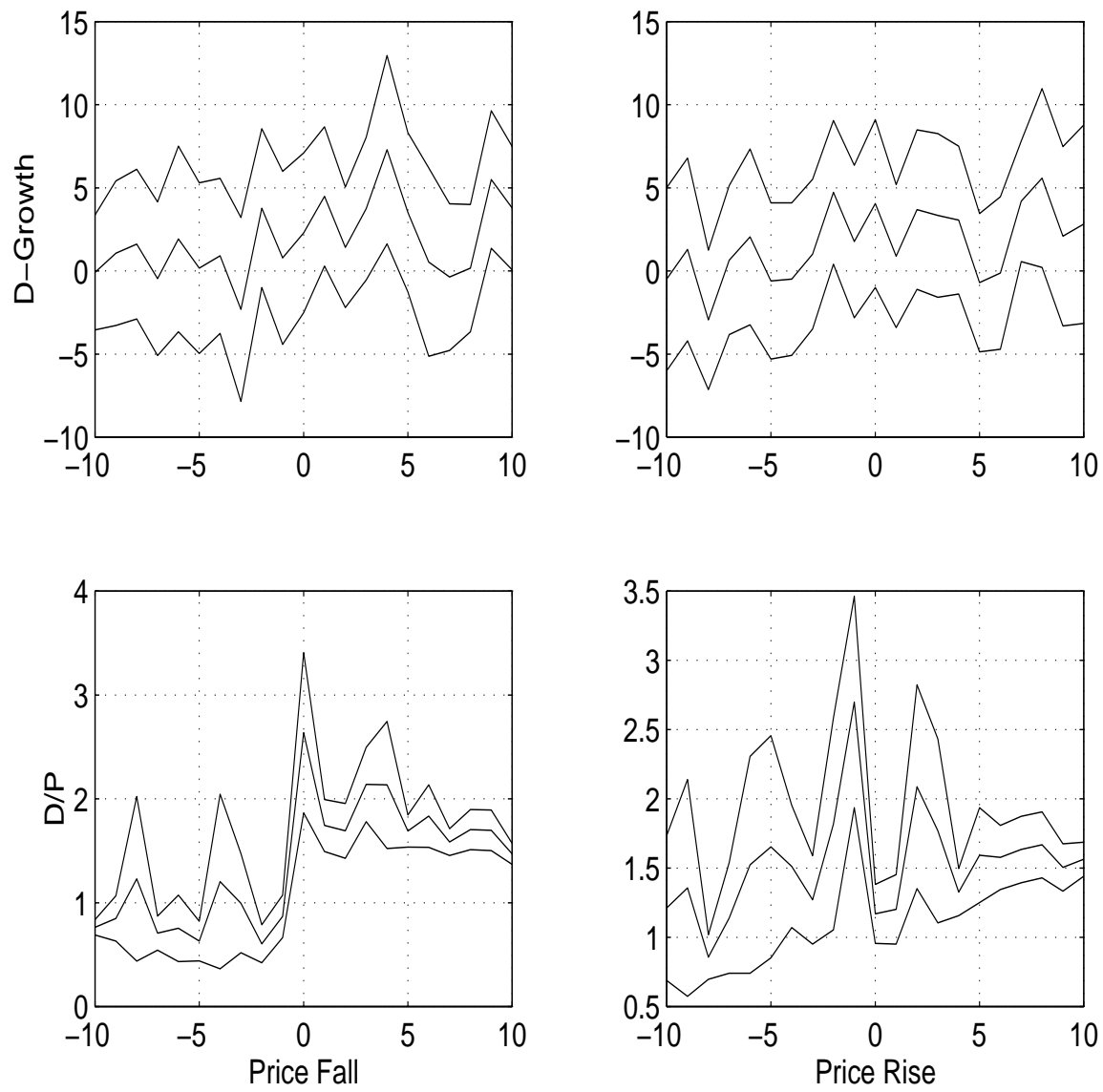


Figure 17: **Price Jumps: Dividend Growth and D/P Ratios**

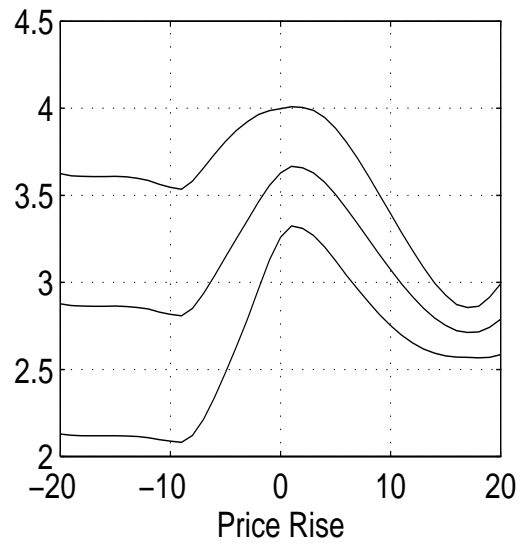
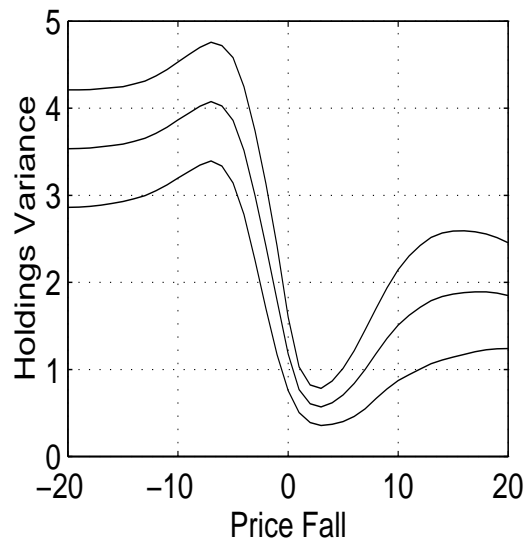
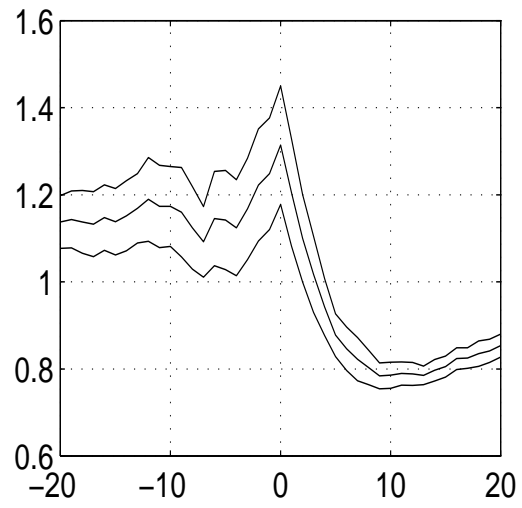
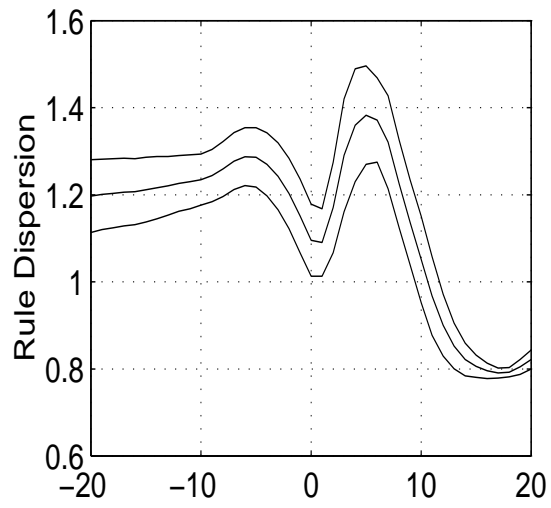


Figure 18: **D/P Extremes: Dispersion and Holdings Variance**

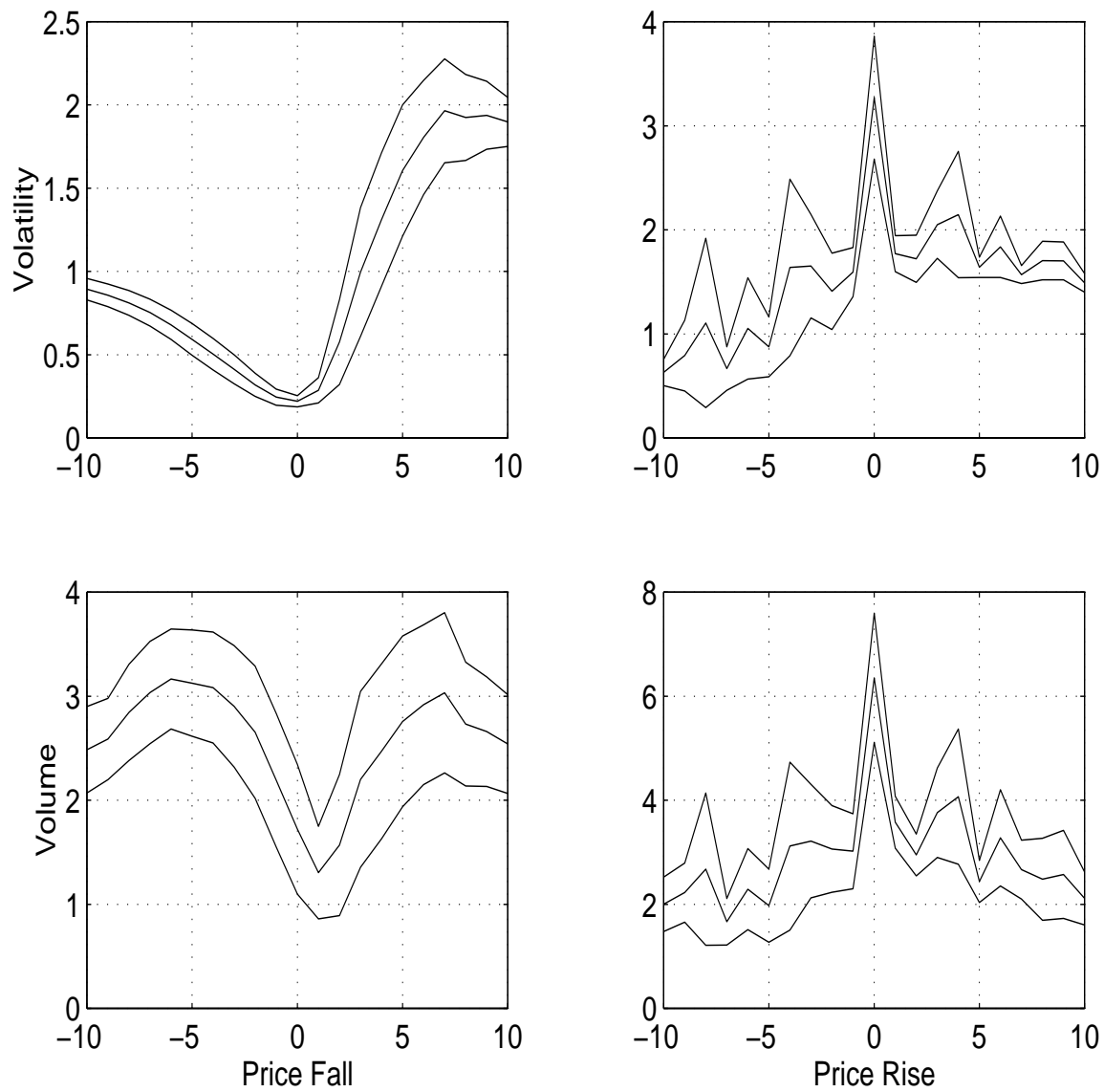


Figure 19: **D/P Extremes: Volatility and Volume**

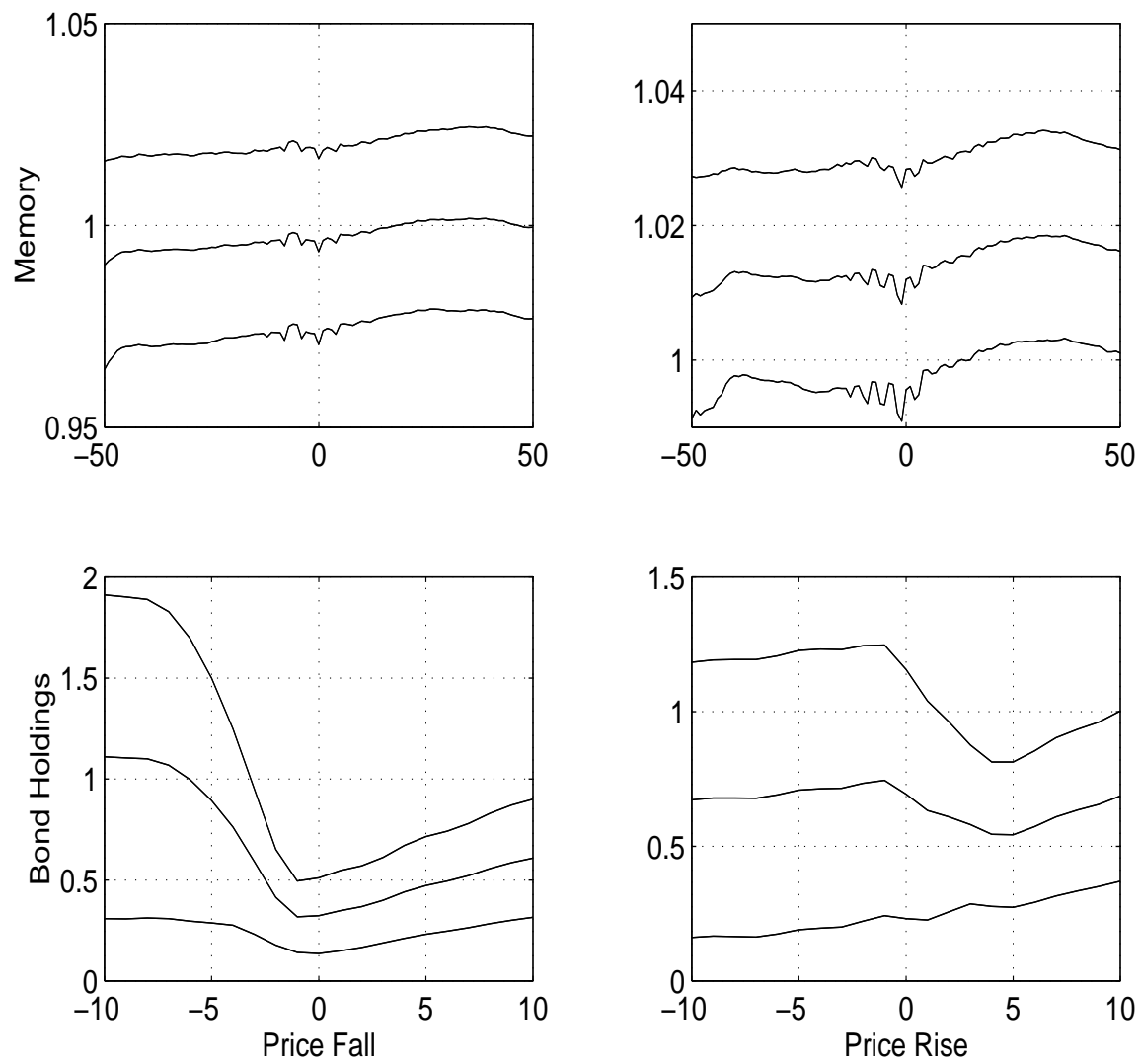


Figure 20: **Wealth Weighted Memory and Mean Bond Holdings**